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ABSTRACT

This monograph is the second in a series of three dealing with the curriculum and evaluation standards for grades K-4 as promulgated by the National Council of Teachers of Mathematics (NCTM). The publication of the standards marks a new guidepost in the evolution of mathematics education and signals the impact of the information society upon all citizens through the recognition of the importance of the mathematical skills involving problem solving, estimation, reasonableness of computed results, and use of technology among others. Included in this second volume are: (1) introductory information concerning the summary changes in the K-4 curriculum as well as essays pertaining to the sequencing and inherent richness of mathematical activities; (2) sample problems and activities dealing with Standard #6, number sense and numeration; (3) activities revolving around Standard #7, whole number operations; (4) sample problems and activities concerning Standard #8, whole number computations; (5) reflections and activities involving Standard #9, geometry and spatial sense; (6) additional activities designed for an independent mathematics center with calculator applications; and (7) appendices that contain the Virginia Council of Teachers of Mathematics position statements pertaining to both calculator usage and testing, and the NCTM position statements on metrication, selection and implementation of instructional materials, and the talented and gifted student. (JJK)

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A publication of the Virginia Council of Teachers of Mathematics

Implementing the Mathematics Standards

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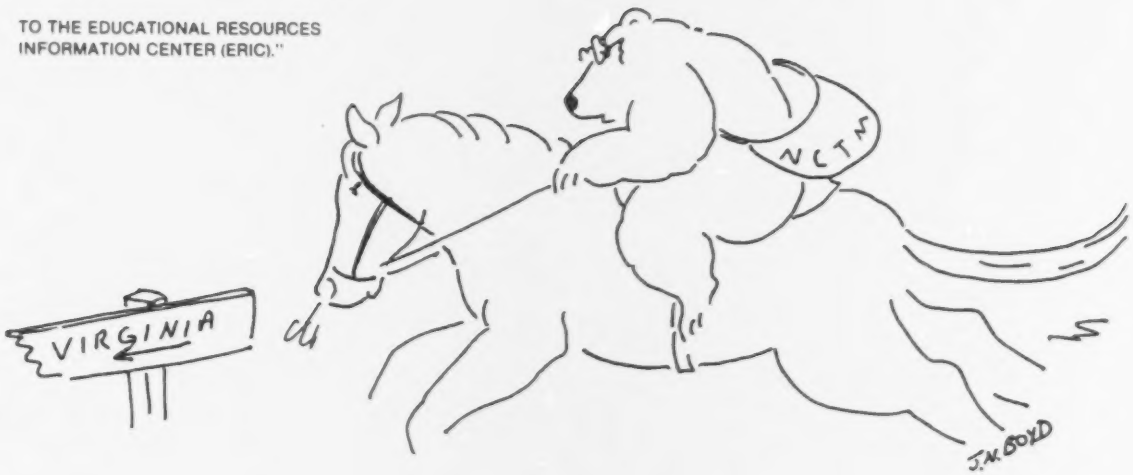

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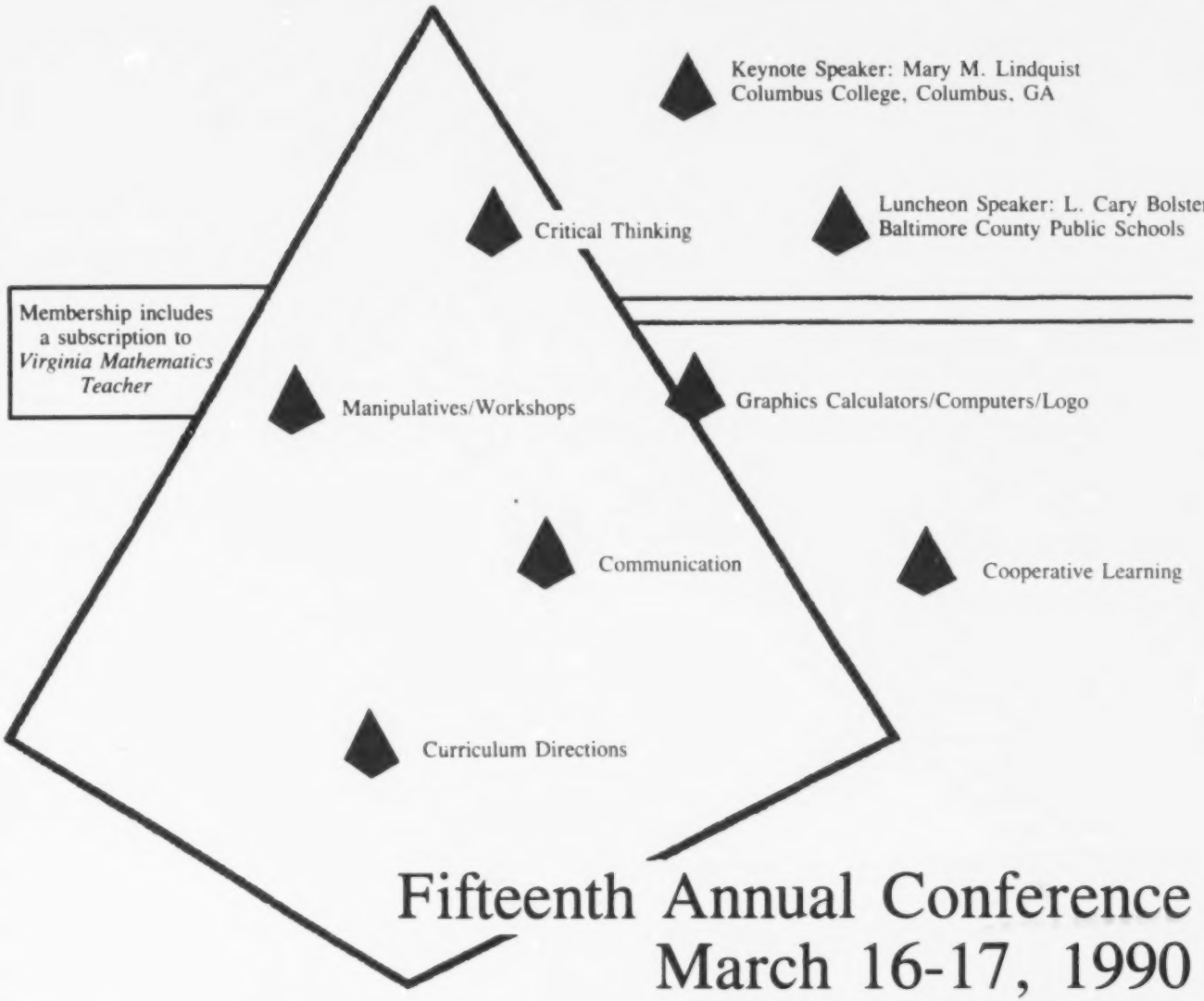

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Volume 2 **Winter 1989-90**

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Prologue

This monograph is the second in a series of three which are being published by the Virginia Council of Teachers of Mathematics this year. Our audience is teachers, principals and others in the Commonwealth who are interested in the mathematical development of young children. It is the hope of the Council that these monographs will offer many teaching ideas as well as serving as a catalyst for much discussion and reflection.

Standards 6 through 9 are explored in this monograph, and Standards 10 through 13 will be presented in the third, and final, monograph.

Once again, I'd like to express our gratitude to authors and to Jim N. Boyd whose contributions appear in this volume. Because of the cooperation of state journals around the country and of the National Council of Teachers of Mathematics, we are able to bring articles and activities which have appeared in their publications. These have been liberally accompanied by Jim's bear art.

We of VCTM believe that our organization, the National Council of Teachers of Mathematics and the regional Virginia NCTM affiliate organizations have your interests and concerns at heart and exist to serve you. Please join us. We will publish a list of regional affiliate groups and contact people in our third and final monograph.

Bob Underhill, Editor
On Behalf of VCTM

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SUMMARY OF CHANGES IN CONTENT AND EMPHASIS IN K—4 MATHEMATICS

INCREASED ATTENTION

NUMBER
- Number sense
- Place-value concepts
- Meaning of fractions and decimals
- Estimation of quantities

OPERATIONS AND COMPUTATION
- Meaning of operations
- Operation sense
- Mental computation
- Estimation and the reasonableness of answers
- Selection of an appropriate computational method
- Use of calculators for complex computation
- Thinking strategies for basic facts

GEOMETRY AND MEASUREMENT
- Properties of geometric figures
- Geometric relationships
- Spatial sense
- Process of measuring
- Concepts related to units of measurement
- Actual measuring
- Estimation of measurements
- Use of measurement and geometry ideas throughout the curriculum

PROBABILITY AND STATISTICS
- Collection and organization of data
- Exploration of chance

PATTERNS AND RELATIONSHIPS
- Pattern recognition and description
- Use of variables to express relationships

PROBLEM SOLVING
- Word problems with a variety of structures
- Use of everyday problems
- Applications
- Study of patterns and relationships
- Problem-solving strategies

INSTRUCTIONAL PRACTICES
- Use of manipulative materials
- Cooperative work
- Discussion of mathematics
- Questioning
- Justification of thinking
- Writing about mathematics
- Problem-solving approach to instruction
- Content integration
- Use of calculators and computers

DECREASED ATTENTION

NUMBER
- Early attention to reading, writing, and ordering numbers symbolically

OPERATIONS AND COMPUTATION
- Complex paper-and-pencil computations
- Isolated treatment of paper-and-pencil computations
- Addition and subtraction without renaming
- Isolated treatment of division facts
- Long division
- Long division without remainders
- Paper-and-pencil fraction computation
- Use of rounding to estimate

GEOMETRY AND MEASUREMENT
- Primary focus on naming geometric figures
- Memorization of equivalencies between units of measurement

PROBLEM SOLVING
- Use of clue words to determine which operation to use

INSTRUCTIONAL PRACTICES
- Rote practice
- Rote memorization of rules
- One answer and one method
- Use of worksheets
- Written practice
- Teaching by telling

Effective Math Instruction in the Primary Grades

Kris Cohoon

Editors Note: While some might take issue with Kris on whether there is a contradiction or not, she does highlight the need for more richness in the math lessons we teach to young learners.

Those who teach math in primary elementary schools are getting two seemingly different messages about appropriate teaching methods: (1) use the Direct Teaching Model (ala Madeline Hunter or Barak Rosenshine); or (2) use developmentally appropriate practices (National Association for the Education of Young Children, NAEYC). The purpose of this article is to help teachers integrate the two types of thinking for effective math instruction.

Direct Teaching Model

Traditionally the focus of conflict is with kindergarten teachers. One school of thought is called the "academic" kindergarten by some and the "pencil and paper" kindergarten by others. The other school of thought is called the "developmental" kindergarten by some and the "play time" kindergarten by others. The academic kindergarten people are presently driven by the direct teaching model which many schools use for their evaluation process. The result in math instruction with its foci on textbook, workbook, and boardwork.

The following are a combined list of steps for direct teaching from Rosenshine and Stevens (1986), Madeline Hunter, and Good and Grouws (1979); (1) anticipatory set; (2) review; (3) presentation of material in small steps, or development, or input and modeling; (4) clear and detailed instructions; (5) guided practice; (6) feedback from all students; (7) feedback and corrections to students; (8) independent practice; and (9) weekly and monthly reviews.

Developmentally Appropriate Practices

The NAEYC (1988) suggested developmentally appropriate practices for students in kindergarten through the third grade. The issue of appropriate practices does not exist only in kindergarten. Developmentally practice does not mean unstructured play, and it does not mean paper and pencil activities of which the contents are measured by standardized tests. It does mean that teachers should plan content in a way to take "optimum advantage of children's natural abilities, interests, and enthusiasm for learning" (NAEYC, 1988 p. 64). The NAEYC proposes the following principles relevant to math instruction:

(1) The math program should enable students to learn math through exploration, discovery, and solving meaningful problems.
(2) Math activities are integrated with other relevant projects in areas such as science and social studies.
(3) Math skills are acquired through spontaneous play, projects, and situations of daily living.
(4) Teacher's manuals are used to guide and structure learning situations and to simulate ideas for interesting math projects.
(5) Math manipulatives are used.
(6) Noncompetitive, impromptu "math stumpers" and number games are played for practice.

No Real Contradiction

No real contradiction exists between the two schools of thought. Direct teaching does not suggest that exploration, discovery, and the use of manipulatives cannot be used during a lesson. In their review of the research on effective math teachers, Romberg and Carpenter (1986) found that effective teachers spend more time on the development part of the lesson. This suggests that teachers can use manipulatives during their lessons. Situations of daily living can be used to develop and apply concepts or provide a schema for the introduction of new material. Developmentally appropriate materials can be used in parts of the direct teaching model.

Teachers need two types of support to teach effective, developmentally appropriate lessons. First, teachers and school administrators must be flexible in their vision of effective instruction. Math instruction should not be confined to a textbook and related workbooks and worksheets. Teachers should not be limited to a forty-five minute period to develop their lessons which is often the case during observations for evaluation. Second, teachers need to be trained to teach using manipulatives and projects in the context of their math curriculum.

References

National Association for the Education of Young Children (1988). NAEYC position statement on developmentally appropriate practice in the primary grades, serving 5- through 8-year olds. *Young Children*, January, 64-84.

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Reprinted from the Kansas *Bulletin*.

Dimensions of Richness in Activities

Bob Underhill

When we learn something new, we usually have knowledge of some aspects. Those aspects are determined by the ways we were either led to encounter the ideas by our teachers and curriculum experiences and tests, or they were determined by the aspects we had to know in order to solve some problem. The problem might be a problem of every day living, or it might be a game-like problem which we want to solve for the fun of it, maybe to satisfy our curiosity, maybe to just have some recreation, maybe to do something enjoyable with another person, or maybe to win in a competitive struggle.

The questions to be answered are these:

1. How can I conceptualize these relationships in my math instruction?
2. How can I focus on these relationships?

Conceptual Richness

There are different ways in which we *know* things. We know some things through our sense of touch; these involve kinesthetic, tactile and motoric skills. We know how to ride a bicycle; we know how to jump three times; we know how a cycle "feels." We also know things through mental imagery. We know what four things look like; we know how to visualize two-thirds of a pie; we know how to visualize 23 as two tens and three ones. We also know things symbolically. We have two major abstract symbol systems, language and mathematics. We know that "three" and "3" are two symbols for a set with three elements. We know that "½" and "one-half" are two symbols for one-half, and we know that "23 + 18 = 41" and "twenty-three plus eighteen equals forty-one" are two symbolic ways to express a sum.

At the most simple level, we are trying to *make connection's* between math symbols, language symbols, doing situations, and pictures or diagrams.

Examples

1. Math symbols → Language Symbols
   Given "3," the children will write "three."

2. Pictures → math symbols
   Given a picture of three elephants, the children will write "3."

3. Situations → Doing
   Given a story about three stooges, the children will place three counters on their desks.

Activities

1. Can you describe activities for the concept of "three" for each of these?
   a. Picture → Manipulative
   b. Language symbols → Situation
   c. Doing → Math symbols
   d. Language symbols → Pictures
   e. Situations → Math symbols
   f. Doing → Situations

Increasing the Richness

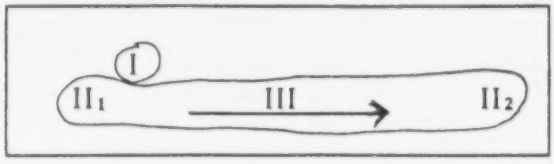
As teachers, we want to make conceptual networks which will tie these ways of knowing together. Here are some aspects:

Mathematical Formats	**Senses**
A. Manipulatives or objects	A'. Talking
B. Pictures or diagrams	B'. Listening
C. Language symbols	C'. Seeing
D. Mathematical symbols	D'. Doing/Feeling/Touching
E. Ethnomathematical stories	E'. Drawing/Writing

Designing Learning Experiences

Think about all of the combinations you can make!

I. Choose a concept such as addition of simple fractions.
II. Choose two or more items in the *left* group (A or B or C or D or E).
III. Choose one item in the *right* group (A' or B' or C' or D' or E').
IV. Create an activity



Example
Concept: Simple fractions
Formats: Manipulatives and pictures
Sense: Talking
Activity: (Manipulatives) talking → (Pictures)

Learners go about the room to six tables where the teacher has placed fraction manipulatives (such as a circular fraction kit with two-thirds) and pictures of various fractional parts. The two learners are to discuss the pictures and decide which pictures are also two-thirds.

Another Example
Concept: Simple subtraction of whole numbers
Formats: Pictures and stories
Sense: Doing/Feeling/Touching
Activity: (Pictures) doing → (Stories)

Learners are presented with some pictures and stories on cards (both on the same card). The learners are asked to show the relationship between each picture and story by using counters.

Another Example
Concept: Square
Formats: Language symbols and pictures
Sense: Seeing
Activity: (Language symbols) seeing → (Pictures)

Given four cards which say "SQUARE," the children will place them next to four pictures on tables around the room which have pictures of square items in them.

Another Example
Concept: Simple multiplication
Formats: Stories and mathematical symbols
Sense: Doing/Feeling/Touching
Activity: (Stories) doing → (Mathematical symbols)

Learners will be given cards with stories and cards with number sentences (e.g., 3x4 = □). They will be asked to use counters to solve the problems and then choose a number sentence card which goes with the story & activity.

Rules-of-Thumb for Sequencing Activities

Bob Underhill

Children bring to the classroom a lot of common sense. Good schooling encourages them to use it. Many people in the mathematics education community are calling this ethnomathematics because it is mathematical knowledge which children acquire simply by being reared in the culture.

A couple simple examples; most children can answer these questions without formal education. A five-year-old when presented with a picture of a mother duck followed by four or five baby ducks can tell you how many baby ducks there are. And nearly every beginning first grader can answer these questions before being taught addition and subtraction:

1. I am giving you three pieces of candy. If I give you two more, how many will you have altogether (assuming there is a collection of candies which can be used).
2. I am giving you four pieces of candy. If you eat three of them, how many will you have left to eat later?

We want children to understand addition, subtraction, multiplication and division of whole numbers, fractions and decimals; measurement; geometry; probability, and so on. Associated with each of these is a set of concepts which have their intuitive origins or correlates in ethnomathematics or mathematics of the child's out-of-school experience. Also, each of these has a set of symbols (e.g., +, -, x, ÷, 5, 2/3, 0.12, =) which can be used to help communicate these ideas, to use them in combinations, and to help solve complex, multi-stepped problems.

It is helpful to think of the following sequence of learning experiences to help children begin with their own intuitive ethnomathematics and to move *as they are ready* to a view of symbolic knowledge which is both rich and meaningful:

1. Readiness (R) Learning Experiences - These challenge learners to use familiar environmental objects in conjunction with their intuitive or ethnomathematical knowledge to solve meaningful story problems *before* mathematical symbols are introduced.
2. Concrete (C) Learning Experiences - These challenge learners to relate their readiness experiences to conventional mathematical symbols (*now* you use the symbols!).
3. Semi-concrete (S) Learning Experiences - These challenge children to substitute pictures and diagrams for real objects in concrete learning experiences.
4. Abstract (A) Learning Experiences - These challenge children to engage in meaningful mathematical activity without the use of concrete or semi-concrete aids.

Here is a chart which summarizes the relationships among the four types of learning experiences:

	R →	C →	S →	A
1. Ethnomathematical stories	X	X	X	X
2. Use of meaningful objects	X	X		
3. Use of pictures and diagrams	(X)		X	
4. Use of mathematical symbols		X	X	X

Instruction which helps children move from R to C to S to A will, if done with sensitivity, greatly enhance the chances that they will develop mathematical knowledge which is rich and meaningful and which they can apply in out-of-school settings.

Certainly a very important question is "At what speed do I try to get learners to move through this sequence?" This is not a simple question. It has four major influences:

1. How old is the child?
2. How developmentally mature is the child?
3. How rich is the ethnomathematical history of the child?
4. How rich is the formal mathematical history of the child?

A *very* lengthy treatise could be written on these influences. Instead, let's briefly explore some rules-of-thumb:

1. *The younger the child, the longer it takes to move through the sequence* (on a given topic, it may take two years, returning to the topic intermittently). For example, a child might follow this overlapping progression when learning addition of whole numbers:
   - Fall of 1st Grade - R→C
   - Spring of 1st Grade - (R→)C→S
   - Fall of 2nd Grade - (R→C→)S→A
   - Spring of 2nd Grade - (C→S→)A

   The items in parentheses are reviews and possible levels of functioning for children who are developing the concept more slowly.

   Another example. A child might encounter the following when learning multiplication of fractions:
   - Fall of 5th Grade - R→C→S
   - Spring of 5th Grade - (R→)C→S→A
   - Fall of 6th Grade - (C→S→)A
   - Spring of 6th Grade - (S→)A
2. *The younger the child, the longer it usually takes on each level of the sequence.* For example, first graders may spend weeks on R and C for addition of whole numbers, but most 5th graders would spend only a few days on R and C for multiplication of fractions.
3. *Every child in grades K-4 needs the opportunity to learn from all four levels.* The very bright and the very mathematically mature may be exposed to the entire sequence in a couple days and never need to return to R or C or even S, but the opportunity is needed to help build the bridges between the common sense of ethnomathematics, childrens iconic knowledge and the formal symbol systems of abstract mathematics.
4. *All children need to work at the level in the sequence at which they are engaged in meaningful mathematical activity.* Meaningful means that they can relate what they are doing to ethnomathematical contexts and iconic knowledge, and can explain in some appropriate way according to their language facility what they are doing and why they are doing it. This explanation must be conceptual (why and how) rather than simply procedural (how).

STANDARD 6:
NUMBER SENSE AND NUMERATION

In grades K–4, the mathematics curriculum should include whole number concepts and skills so that students can—

- ***construct number meanings through real-world experiences and the use of physical materials;***
- ***understand our numeration system by relating counting, grouping, and place-value concepts;***
- ***develop number sense;***
- ***interpret the multiple uses of numbers encountered in the real world.***

Focus

Children must understand numbers if they are to make sense of the ways numbers are used in their everyday world. They need to use numbers to quantify, to identify location, to identify a specific object in a collection, to name, and to measure. Furthermore, an understanding of place value is crucial for later work with number and computation.

Intuition about number relationships helps children make judgments about the reasonableness of computational results and of proposed solutions to numerical problems. Such intuition requires good number sense. Children with good number sense (1) have well-understood number meanings, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitudes of numbers, (4) know the relative effect of operating on numbers, and (5) develop referents for measures of common objects and situations in their environments.

Children come to understand number meanings gradually. To encourage these understandings, teachers can offer classroom experiences in which students first manipulate physical objects and then use their own language to explain their thinking. This active involvement in, and expression of, physical manipulations encourages children to reflect on their actions and to construct their own number meanings. In all situations, work with number symbols should be meaningfully linked to concrete materials. Emphasizing exploratory experiences with numbers that capitalize on the natural insights of children enhances their sense of mathematical competency, enables them to build and extend number relationships, and helps them to develop a link between their world and the world of mathematics.

If children are to develop good number concepts, considerable instructional time must be devoted to number and numeration. Children's experiences with numbers are most beneficial when the numbers have meaning for them. A variety of place-value tasks that assess children's thinking can be used to identify those numbers that have meaning to individual students; traditional numeration tasks are not good indicators of children's understanding. Teachers can also provide exploratory experiences with larger numbers, but symbolic tasks with numbers should not be presented in isolation and should not be emphasized until the numerals have been carefully linked to concrete materials and children understand the major concepts.

STANDARD 6:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Number Sense & Numeration Standard

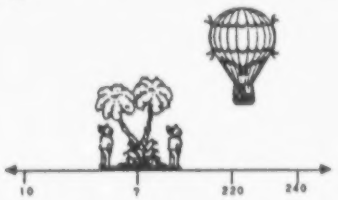
1. Activity "Win a Hundred" on page 49 of *Mathworks*. This activity helps the students to develop an understanding of regrouping. The game can be made more stimulating by having the students make up stories about their money gains and exchanges when they made a transaction with the banker. (Christine Whitley)

2. Activity "So, What's the Number?" on page 397 of *Mathworks*. This activity helps the students to develop number sense. Clues about a particular number are given and the students must use deductive reasoning to figure out what number the clues are describing. (Christine Whitley)

3. Have children make place value kits. Then call out numbers and have them show that number in two different ways using ones and tens in the place value kit. Have the children work in pairs. One child comes up with a number and writes it down. The partner shows the number using the manipulatives and then explains why they used the tens and then the ones. (Elyse Williams)

4. Ask the students to write their names and house numbers (not streets) on a piece of paper. Collect this data. Have the students order the house numbers from lowest to highest. (Mathwork, p. 34) Also, as a variation, have the names covered and let the class guess which house number is a given student's, by the "given study saying...it's higher or lower..." (Brenda Sloss)

5. A game can be used where the students have to guess "where to drop a raft to rescue people off of an island." The picture would look like the following:



   If they pick the correct number, the people would be "rescued" from the island. (Marnea Prestel)

6. Group children with six members in a group. Each group stands in a line and counts off "first, second, third, fourth, fifth, sixth." Have cards such as "third player moves two people backward." Each time a card is drawn, students count off again, since the order changes. (Jeanie Mokwa)

7. Children are given 50 beans. They are each asked to count out 26 in any way they choose. Some children count out 26 beans and some children count out 2 sets of ten and 6 ones. They are asked to discuss their results. (place-value knowledge) (Melissa Baker)

These ideas are from Tech students in the K-8 certification program.

FOR YOUR CLASSROOM: ELEMENTARY MENTAL GYMNASTICS

Sister Mary Petronia Van Straten
Mount Mary College
Milwaukee, Wisconsin

Many children do not work to capacity when it comes to performing computations mentally. I heard a speaker once remark that they suffer from pencilitis, meaning of course that as soon as they are given a computation to do, they grab their pencils. Be that as it may, we as teachers should provide opportunities for children in this regard.

Let me cite a few ways in which teachers could do this. Computations like 7 x 15 or 8 x 17 provide good examples. No pencils, please! Children should be taught to think of 15 as 10 + 5; then to visualize 7 x (10 + 5); then to use the distributive principle and take 7 x 10 and 7 x 5 and add the products, 70 + 35 = 105.

For the second example, the thought process would be as follows:

$$\begin{aligned} 8 \times 17 &= 8 \times (10 + 7) \\ &= (8 \times 10) + (8 \times 7) \\ &= 80 + 56 \\ &= 136 \end{aligned}$$

Giving children practice for a short period each day for a week, teachers might be amazed at the results. Children can do it! Try it to prove it to yourself.

Another source would be like examples given below. The difficulty and length of each chain of operations would depend upon grade level and/or upon which facts or operations a teacher was endeavoring to provide practice for the children.

A teacher might state what is to be done orally and have each child in the class write the final answer for each chain of operations. After ten are given and the children have written their responses, the correct answers may be given. This is one way of proceeding. It is surely better than having the teacher state one and then call on one child for the answer. Often, this does not not make every child in the class think through the process.

What I think is definitely a better way, and I have tried it, is to have the chains of operations put on a tape. The teacher has each student take a paper and number up to ten. The teacher then turns on the tape, and the children begin. This provides the children with the opportunity to listen carefully and to improve their power of concentration, two skills they very much need to acquire. After completing ten chains, the children are given the correct responses. If a tape is used the teacher can regulate the pace at which the examples are given. The tape also provides a means of going back to a chain, should there be an argument about a right answer.

The chains of operations might also be placed on transparencies.

SIMPLE EXAMPLES OF CHAINS OF OPERATIONS

#1	Start with 4. Multiply by 7. Add 3.	#2	Start with 6. Multiply by 5. Subtract 5.	#3	Start with 7. Multiply by 8. Add 7.
#4	Start with 9. Multiply by 9. Subtract 1.	#5	Start with 8. Multiply by 8. Add 6.	#6	Start with 7. Multiply by 6. Add 3.
#7	Start with 8. Multiply by 9. Add 4.	#8	Start with 7. Multiply by 7. Add 5.	#9	Start with 8. Multiply by 6. Add 4.
#10	Start with 9. Multiply by 7. Subtract 3.	#11	Start with 8. Multiply by 0. Add 13.	#12	Start with 9. Multiply by 6. Subtract 6.

As children become more apt at the procedure and the process, teachers should challenge them with longer and more difficult chains. Examples are given below.

#13 Start with 4; add 6; divide by 5; multiply by 9; divided by 3, add 7.

#14 Start with 5; multiply by 5; subtract 8; add 3; divide by 4; multiply by 0; add 15; subtract 7; add 9.

#15 Start with 3; multiply by 7; subtract 9; divide by 6; multiply by 10; subtract 6; add 8; subtract 6; divide by 4; subtract 4.

#16 Start with 8; multiply by 9; subtract 8; divide by 2; subtract 5; divide by 9; add 18; divide by 3; multiply by 8; subtract 6.

#17 Start with 9; multiply by 7; subtract 8; divide by 5; add 13; divide by 6; multiply by 7; add 2; divide by 6; subtract 5.

#18 Start with 7; multiply by 7; subtract 9; divide by 5; multiply by 8; subtract 8; divide by 8; multiply by 6; divide by 7; add 17.

#19 Start with the first odd number after 17; subtract 5; take 1/2 of your answer; add 5; subtract 4; take 3/4 of your answer; add 1/4; convert this number to an improper fraction.

#20 Start with 10; multiply by 5; take 10 percent of your answer; divide by 5; add 11; subtract 4; multiply by 9; add 28; take 25 percent of your answer.

BUZZ BANG

Adapted from MATH FUN AND GAMES
by Marilyn Rehwald

This math activity is designed to give pupils practice in recognizing multiples of numbers. At the same time, pupils consider it a fun game!

OBJECTIVES: After participating in this activity, TSSBAT:

1. Recognize multiples of the numbers 2 - 9.
2. Recognize common multiples of number pairs.
3. Demonstrate increased speed and accuracy in mental arithmetic.

PROCEDURE:

Write any two numbers on the chalkboard, eg. 2 and 5. Be certain pupils have learned the multiples of these numbers ahead of time. Assign the word "buzz" to the first number and the word "bang" to the second number.

Buzz = 2 or any multiple of 2. Bang = 5 or any multiple of 5.

Have all pupils stand. Teacher begins by counting "1." Pupils continue counting as each gets a turn. If a pupil's number is 2 or any multiple of 2, he should say "buzz" instead of the number. If he says the number by mistake, he is out of the game and must sit down. 2, 4, 6, 8, etc. are all "buzz."

If a pupil's number is 5 or any multiple of it, he should say "bang" instead of the number. If he says the number by mistake, he is out of the game and must sit down. 5, 15, 25, etc. are all "bang."

If a pupil's number is a common multiple of both 2 and 5, he should say "buzz-bang" instead of the number. If he says anything else, he is out of the game. 10, 20, 30, etc. are all "buzz-bang."

Play continues around the room until only the winning pupil is left standing.

Here is how the first part of a game would sound if done correctly for numbers 2 and 5:

1, buzz, 3 buzz, bang, buzz, 7, buzz, 9, buzz-bang, 11, buzz, 13, buzz, bang, buzz, etc.

After pupils have mastered this simple form of the game, you may gradually make it more difficult by using other numbers.

The book cited above is highly recommended by COUNT MATHULA as an excellent source of math games and other activities.

COUNT MATHULA is A. R. Balian

Mental Math Activities

Sally Syren

One of the five major areas of emphases in the Mathematics Framework is estimation and mental arithmetic. The systematic teaching of estimation and mental arithmetic as an integral part of the daily program will improve students' number sense, problem solving skills and ability to logically reason.

Using mental arithmetic allows students to use approaches other than paper and pencil to make calculations. One way to incorporate mental arithmetic into daily planning is through the use of 5-minute warm-up drill activities.

Included below are several possible activities using the strategies of counting on, counting backwards, patterns, compensation, using equivalent forms, using different operations, and making choices.

Activity 1: Human Patterns
Materials: None
Directions: Divide the class into groups of four. Let groups decide on a pattern such as stand/stoop, stand/stoop, or hands on head/hands on hips, hands on head/hands on hips. Let each group form a pattern and have other groups study the pattern and copy it.

Activity 2: What's the number?
Materials: Each child has one of the number cards 13-31
Directions: Leader calls out a number with a description. Example: The number is one more than 15. Student with card 16 stands. Other examples may be: the number 1 less than 27 (26); the number between 19 and 21 (20).

Activity 3: Combinations
Materials: Cards labeled 1¢ to 20¢, coins (pennies, nickels, dimes)
Directions: Have students work in pairs. One student holds up a card. The other student calls out a combination of coins that equals the value shown on the card. Then both check the answer by counting the coins. Students then reverse roles.

Activity 4: What's In the Bag?
Materials: paper bag, counters
Directions: Place 5 counters in a bag. Show students 1 counter. Tell students that there are 6 counters in all, and ask them how many counters are in the bag. Do several examples with sums to 6 or 8. Students may work in pairs with one placing the counters in the bag and the other giving the answer.

Activity 5: Skip Counting
Materials: none
Directions: Call out a sequence of numbers such as 3, 6, 9, 12. Ask students to name your pattern. Repeat again leaving out a number: 3, 6, 12. Have students name the missing number.

Activity 6: Adjusting
Materials: none
Directions: Show 30 + 60 on the chalkboard and discuss that it is easy to add multiples of 10. Now write 31 + 59. Show that adjusting by adding 1 to 59 and taking away 1 from 31 is much easier to add.

31	1 less	30
+59	1 more	+60
90		90

Activity 7: Elegant Eights, Nice Nines
Materials: None
Directions: Call on students to find the difference of 13 - 8. (5) Then write on the chalkboard the following examples. Discuss the rule of 8 or 9.
When subtracting 8, first subtract 10, then add 2. Also discuss the pattern.

23	33	43	53	63	93
- 8	- 8	- 8	- 8	- 8	- 8

Activity 8: Sum + Sum
Materials: Calculator (optional)
Directions: Tell students that this mental math drill will include basic facts plus memory. Give students a basic fact, such as 3 + 4. Tell students to hold this sum in their heads. Give students another basic fact, such as 2 + 1. Ask students to add the sum of the first fact to the sum of the second fact, 7 + 3 = 10. As students become proficient with two sums, add a third and so on. Ask a recorder to keep a running total of the sum of the facts on a calculator.

Activity 9: Thumbs Up
Materials None
Directions: Write the number 46 on the chalkboard. Then give the following list of expressions to the students and ask if they are another name for the number 46. If it is, students put their thumbs up, if not thumbs down.
(a) 46 ones (thumbs up)
(b) 3 tens, 16 ones (thumbs up)
(c) 1 ten, 26 ones (thumbs down)
(d) 4 tens, 6 ones (thumbs up)

Sally wrote this for the ComMuniCator (California).

STANDARD 7:
CONCEPTS OF WHOLE NUMBER OPERATIONS

In grades K–4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can—

- ***develop meaning for the operations by modeling and discussing a rich variety of problem situations;***
- ***relate the mathematical language and symbolism of operations to problem situations and informal language;***
- ***recognize that a wide variety of problem structures can be represented by a single operation;***
- ***develop operation sense.***

Focus

Understanding the fundamental operations of addition, subtraction, multiplication, and division is central to knowing mathematics. One essential component of what it means to understand an operation is recognizing conditions in real-world situations that indicate that the operation would be useful in those situations. Other components include building an awareness of models and the properties of an operation, seeing relationships among operations, and acquiring insight into the effects of an operation on a pair of numbers. These four components are aspects of *operation sense*. Children with good operation sense are able to apply operations meaningfully and with flexibility. Operation sense interacts with number sense and enables students to make thoughtful decisions about the reasonableness of results. Furthermore, operation sense provides a framework for the conceptual development of mental and written computational procedures.

Instruction on the meaning of operations focuses on concepts and relationships rather than on computation, which is the focus of Standard 8. Children need extensive informal experience with problem situations and language prior to explicit instruction and symbolic work with the operations. Thus, informal experiences with all four operations should begin in kindergarten and continue through grade 4. Instruction should help children connect their intuitions and informal language to operations, including the mathematical language and symbols of each operation. Children should encounter the four basic operations in a wide variety of problem structures. For example, in addition to problems with joining and separating structures, teachers should provide problems involving comparing and equalizing. Time devoted to conceptual development provides meaning and context to subsequent work on computational skills.

STANDARD 7:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

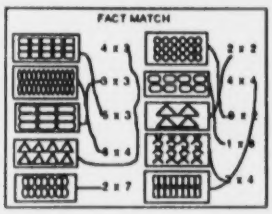
Activities for the Whole Number Operations Standard

1. Children look at the scale, and numbers on the scale to decide when operation is needed to make the scale balance. (Stephanie Siebeking)

2. Have construction paper and cut outs for visual aids, of two hamburgers and ketchup topping, and a mustard topping, and a mayonnaise topping. Have students generate all the possible topping combinations. Give students a variety of other situation problems to work on. (Eileen Tully)

3. I would have 50 or so cards written up with additional subtraction, multiplication and division problems on them. The children must solve the problem and write the number of the card and the equation on their paper. They will do this for a while and then they will get with partners. Each group will have a set of cards with numbers on them. They will select 2 cards and write any type of story problem that they want to, they will then trade and solve each others problems. In this way, children must make up as well as solve problems. (Lisa Potvin)

4. Have a bulletin board "Fact Match" to illustrate the concept of multiplication. Have the students either write the math fact or match the arrays to multiplication facts using yarn.



(Brenda Sloss)

5. If Susan has 3 shirts, 4 pairs of pants and 2 skirts, how many outfits can she make - using a shirt and a pair of pants or a shirt and a skirt? (Karen Fallin)

6. You have to color a picture of a little girl who is wearing a blouse and a skirt. Using the 4 crayons you have, how many different outfits can you make the girl have? (Kay Dillon)

7. Each pair of students will be given an equation. From this, the students will write a story problem. The teacher will collect the story problems and either redistribute the problems for a different pair of students to solve from the story problem, or the teacher will read each story problem out loud and the students will solve the story problems.

These ideas are from Tech students in the K-8 certification program.

STANDARD 8:
WHOLE NUMBER COMPUTATION

In grades K–4, the mathematics curriculum should develop whole number computation so that students can—

- ***model, explain, and develop reasonable proficiency with basic facts and algorithms;***
- ***use a variety of mental computation and estimation techniques;***
- ***use calculators in appropriate computational situations;***
- ***select and use computation techniques appropriate to specific problems and determine whether the results are reasonable.***

Focus

The purpose of computation is to solve problems. Thus, although computation is important in mathematics and in daily life, our technological age requires us to rethink how computation is done today. Almost all complex computation today is done by calculators and computers. In many daily situations, answers are computed mentally or estimates are sufficient, and paper-and-pencil algorithms are useful when the computation is reasonably straightforward. This standard addresses the importance of teaching children a variety of ways to compute, as well as the usefulness of calculators in solving problems containing large numbers or requiring complex computations. Related to this goal is the necessity of having reasonable expectations for proficiency with paper-and-pencil computation. Clearly, paper-and-pencil computation cannot continue to dominate the curriculum or there will be insufficient time for children to learn other, more important mathematics they need now and in the future.

By emphasizing underlying concepts, using physical materials to model procedures, linking the manipulation of materials to the steps of the procedures, and developing thinking patterns, teachers can help children master basic facts and algorithms and understand their usefulness and relevance to daily situations. This approach also promotes efficient learning of computational techniques and furthers the development of children's reasoning, mathematical insight, and confidence in their ability to do mathematics. Instruction also should emphasize a variety of ways to compute, the importance of checking whether computed results are reasonable, and the need to make appropriate decisions about how to compute in a problem situation. An awareness that computation is learned and used to attain some goal develops when problem situations and computations are explicitly linked throughout all aspects of work with computations.

STANDARD 8:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Whole Number Computation Standard

1. Tell the children we are planning a _____ party. What are some things we would need to buy. Create a list on the board. Show quantity needed of each and a _____ price which teacher creates. After planning is done, pose the question, "Approximately how much money are we spending on our party?" Each child uses estimation to come up with an approximate answer. Discuss if our estimation is close to being accurate and what would make it off. (Stephanie Siebeking)

2. Activity "Right On!" on page 351 of *Mathworks*. In this activity the students use calculators to perform computational addition strategies. Develop a winning strategy for the game NIM where there are 2 players. The loser is the first person to subtract an amount that causes the total to equal 0. The game begins with the number 31 and each player must subtract 4, 3, 2, or 1 - they say what 31 - 4, 3, 2, or 1 is. (Christine Whitley)

3. Have students create menus for their own restaurants. Then create questions based on their menus. Have them estimate and calculate their problems. (Marnea Prestel)

4. Give students grocery ads and a total amount of money (say $25.00). Have students go through and make a grocery list. The students are to estimate how much they spent on items but can't exceed $25.00. Lastly, give students a calculator to see the actual total their bill will come to. (Jennifer Hoelk)

5. A Learning Center/Dramatic Play area is set up with an adding machine and several fake food items with prices. Students pick out the food they want and estimate the total. The cashier "rings up" the total on the adding machine, and they can see how close they come. (Jeanie Mohwa)

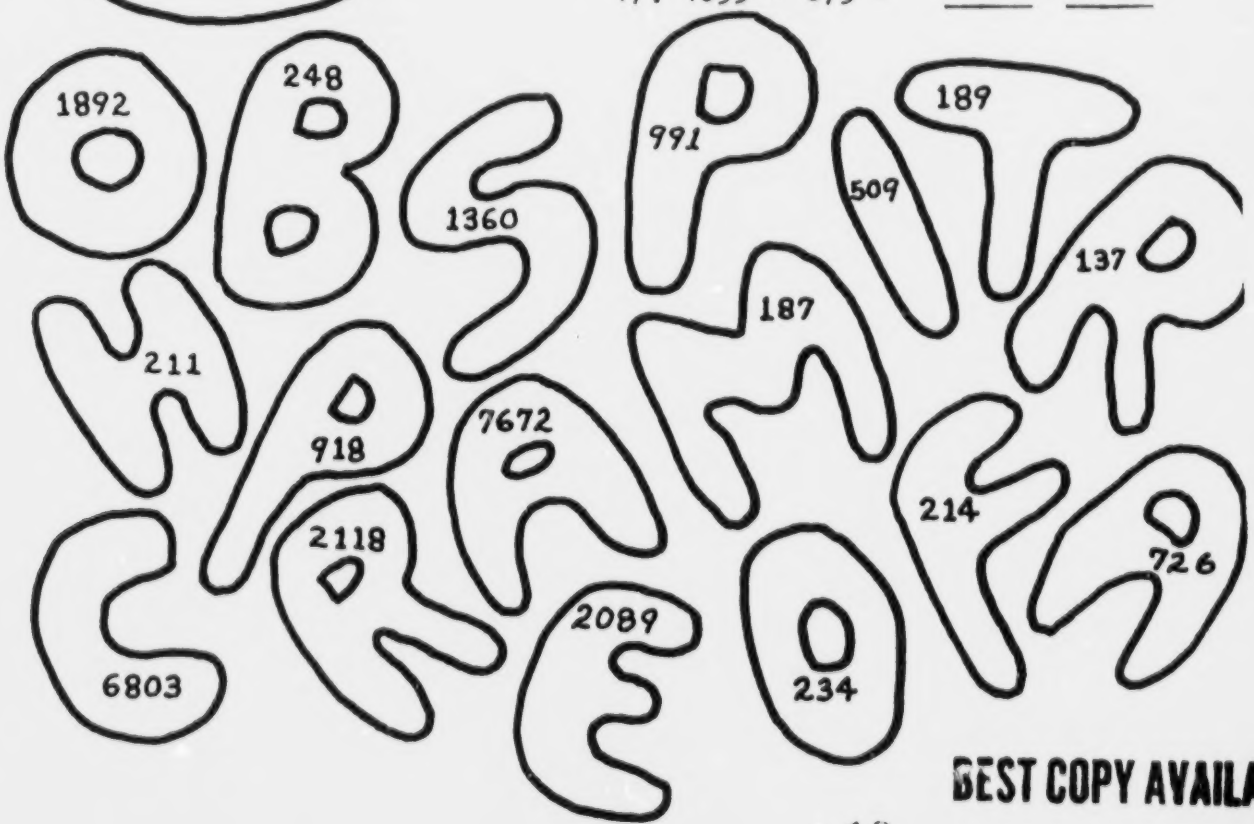
6. -collect "Sale" circulars from a local department store
   -have students work in groups, 6 groups total
   -2 of the 6 groups are given calculators to use; 2 of the 6 groups are told to use paper and pencil to add; and, the last two groups are asked to estimate.
   -each group can spend no more than $25.00 on the sale items.
   -each group calculates by their method given (calculator, addition or estimation) what they can and will buy
   -groups report their "shopping list" to the rest of the class and explain why they can afford those items listed
   -students compare findings and methods of addition
   (Carolyn Croy)

These ideas are from Tech students in the K-8 certification program.

DRESS THE SNOWMAN

Complete the problems and find the corresponding Mod Letter below with the correct answer. Dress the snowman with the articles spelled out.

		Letter:
ARTICLE 1.		
1. 631 - 420 =	____	____
2. 865 - 139 =	____	____
3. 711 - 522 =	____	____
ARTICLE 2.		
4. 5613 - 4695 =	____	____
5. 1416 - 907 =	____	____
6. 2643 - 1652 =	____	____
7. 5831 - 3742 =	____	____
ARTICLE 3.		
8. 683 - 435 =	____	____
9. 519 - 382 =	____	____
10. 5007 - 3115 =	____	____
11. 316 - 82 =	____	____
12. 713 - 526 =	____	____
ARTICLE 4.		
13. 9631 - 8271 =	____	____
14. 7533 - 730 =	____	____
15. 8644 - 972 =	____	____
16. 6531 - 4413 =	____	____
17. 1093 - 879 =	____	____



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Let's Use Calculators

By **David R. O'Neil** *and* **Rosalie Jensen,** *Georgia State University, Atlanta, Georgia*

This month the activities center around the third recommendation for the curriculum of the 1980s, namely, that "mathematics programs take full advantage of the power of calculators and computers at all grade levels." Specifically, we will suggest how an elementary school mathematics teacher might use the calculator to help students—

a. enrich their understanding of place value;
b. reinforce their knowledge of the basic facts;
c. develop an appreciation of the inverse relationships between addition and subtraction, and between multiplication and division;
d. acquire an understanding of the properties of commutativity, associativity, and distributivity (multiplication over addition);
e. practice estimation skills.

Calculators have an important role to play in the elementary school mathematics classroom. They are readily available (for under $10), are durable, and can be used every day. They can provide immediate feedback to the student, which often results in increased student confidence. Additionally, the decreased time required for "tedious" computations allows the teacher to explore more topics with students.

With these advantages in mind, we have selected some activities to assist the classroom teacher in effectively employing the calculator in the elementary school classroom. In the activities that follow, we are assuming that students are knowledgeable about the calculator to the extent that they are familiar with the keyboard layout and know how to press the keys (in the proper order) to obtain an answer to a specified, simple operation. We recommend that a calculator with an 8-digit display and the four fundamental operations be used.

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Place Value

One of the most important mathematical concepts to be learned in the elementary grades is that of place value. In the decimal system of numeration, students must know that the value of any digit in a numeral depends on its position in that numeral. Students should also understand the relationship between the standard or compact form and the expanded notation of any numeral; that is, that 432 = 400 + 30 + 2. Activities one and two are ways of using the calculator to reinforce this concept. Activity three may be used to practice adding, subtracting, multiplying, or dividing, using powers of ten.

Activity one

Materials needed

A calculator for every two students is needed.

A set of fifteen to twenty cards with two- and three-digit numerals written in the expanded form.

For example:

$30 + 5 = \square$
$50 + 6 = \square$
$200 + 40 + 7 = \square$

As children become more confident, more challenging exercises can be given.

For example:

$300 + 5 = \square$
$1000 + 50 = \square$

Instructions

1. Shuffle the cards and place them face down.

2. Turn over a card and lay it face up where both students can see it.

3. Both students try to write the standard or compact name for the number represented. Player one must use the calculator to find the indicated sum. Player two must use paper and pencil, but not the calculator.

As children do exercises like these, they should also read some of the numbers out loud, to be sure that they see the relationship between the expanded form and the way in which the numerals are read. For example, the sum of 200 + 40 + 6 is read "two hundred forty-six."

Activity two

Materials needed

A calculator for every student.

A set of twenty to thirty cards with examples like the following:

$30 + 6 =$ ____
$25 =$ ____ + ____
$100 + 40 + 6 =$ ____
$324 =$ ____ + ____ + ____

The exercises in this set go from the standard notation to the expanded

form, and from the expanded form to the standard notation.

Instructions

1. Shuffle the cards and place them face down.

2. Turn over a card and write the answer to the problem.

3. Check your answer, using the calculator.

Activity three

Materials needed

A calculator for every two students.

A set of cards with examples involving powers of ten in the four basic operations.

For example:

36 + 10 = ___	47 − 10 = ___
23 × 10 = ___	60 ÷ 10 = ___
348 + 100 = ___	369 − 100 = ___
42 × 100 = ___	7000 ÷ 100 = ___
456 + 10 = ___	456 + 100 = ___
456 − 10 = ___	456 − 100 = ___
456 × 10 = ___	456 × 100 = ___
456 ÷ 10 = ___	456 ÷ 100 = ___

Instructions

The instructions are the same as for the first activity.

Activities one through three can easily be extended to include larger numbers or decimal fractions by changing the cards used. Talk with the children about what is happening in these exercises. What patterns do they see? Encourage them to make up problems and to do some exploring with other numbers. For example, you might have them multiply a number by 10, then 100, then 1000, and so on. You might have them put a number like 356 on their calculator. Then ask them how they could change the number to 346.

Computation

Calculators can also be used effectively to provide students with motivated and competitive practice on basic facts and the computational algorithms. Activity four is an individual activity wherein a student can use the calculator to practice the basic multiplication or addition facts. Special care should be taken to ensure that the numbers on the cards do not require more than the designated single-digit input. For example, 11 and 13 could not be represented as the product of two single-digit numbers. (Use only numbers that occur in the product section of a multiplication fact chart.) This activity could be used to drill on the addition facts if only the numbers 0 through 18 are used. In either case, for some sums or products, multiple answers are possible.

For example:

$$\begin{aligned} 24 &= 3 \times 8 \\ &= 4 \times 6 \end{aligned}$$

and

$$\begin{aligned} 12 &= 3 + 9 \\ &= 4 + 8 \\ &= 5 + 7 \\ &= 6 + 6 \end{aligned}$$

and so on.

Activities five through seven are designed to focus on the commutative, associative, and distributive properties, respectively. These activities are most effective when used with two teams of students, although individuals may also benefit from them. If teams are used, the calculators should be passed among the members of the team after each problem.

It is important that sets A and B for these activities are in matched pairs. The numbers in the right-hand corner of the cards will enable you or the student to order the cards easily.

Activity four

Materials needed

A calculator for every student.

A set of cards with the basic-fact sums or products written on them.

For example:

Products like 27, 36, 49, 64, 42, and so on.

Sums like 8, 11, 5, 15, 9, and so on.

Instructions

1. Shuffle either the sum or product cards and place them face down.

2. Turn over a card.

3. Pressing *only* single digits, an operation sign (+ or ×), and the equal sign, use the calculator to show the number on the card. For example, for the number 27 you could press 3 ⊠ 9 ⊟ .

4. If you are correct, place the card in a separate pile and turn over another card.

5. If you are incorrect, place the card on the bottom of the face-down deck.

Activity five

Materials needed

A calculator for every two students.

Two sets of ten to fifteen cards on which addition (or multiplication) expressions are written. Note that set A differs from set B only in that the addends (or factors) are commuted. Number the cards for easier reference.

For example:

	Set A
(1)	25 + 16 = ___
(2)	12 + 8 = ___
(1)	9 × 8 = ___
(2)	6 × 7 = ___
	Set B
(1)	16 + 25 = ___
(2)	8 + 12 = ___
(1)	8 × 9 = ___
(2)	7 × 6 = ___

Instructions

1. Each player is given a set of cards; one gets set A and the other, set B.

2. Tell the players to pull card number 1 (or any other number) from their set and then calculate the answer. After several examples have been done, have the children compare their results and the problems. Continue this procedure until the majority of the class understands the property being demonstrated.

Activity six

The exercises in this activity illustrate

the associative property with both addition and multiplication. Prior to doing these exercises, children will need to know that the parentheses in an expression means to do the operations inside the parentheses first.

Materials needed

A calculator for every two students.

Two sets of ten to fifteen cards, each with addition (or multiplication) expressions involving the associative property. Sets A and B are the same except for the grouping of operations.

For example:

Set A

(1) $(12 + 16) + 18 =$ ___

(2) $(6 + 28) + 14 =$ ___

Set B

(1) $12 + (16 + 18) =$ ___

(2) $6 + (28 + 14) =$ ___

Instructions

The instructions are the same as for activity five.

Activity seven

In this activity, as in the preceding one, the children must know how the parentheses affect the sequence of operations.

Materials needed

A calculator for every two students.

Two sets of ten to fifteen cards on which, respectively, the two forms of the distributive property for multiplication over addition have been written.

For example:

Set A

(1) $3(4 + 5) =$ ___

(2) $6(3 + 8) =$ ___

Set B

(1) $(3 \times 4) + (3 \times 5) =$ ___

(2) $(6 \times 3) + (6 \times 8) =$ ___

Instructions

The instructions are the same as for the two preceding activities.

Estimation

If the calculator and computer change the way individuals determine numerical answers, as many suggest, it is incumbent upon us as teachers to provide more and better instruction on estimation. Calculators and computers will perform operations on numbers accurately, but the answer that the calculator gives may not be the right answer to the given problem. The problem doer may have selected the wrong operation, pressed the wrong key—possibilities for errors still abound. The successful problem solver needs to know whether the answer he or she gets is a reasonable answer. Every answer should be estimated as well as calculated.

An "educated guess" requires a special skill that we believe can be developed only through repeated practice. Teaching estimation must begin early in the elementary school and continue throughout the elementary and secondary grades. The next two activities suggest two ways in which the calculator can be used to provide practice in estimation.

Many children may need some guidance as to how to go about making a guess. You may find it helpful to suggest that they might try a series of questions: Is the answer more or less than 10? more or less than 100? more or less than 1000? and so on. Once they have decided whether the answer is a one-digit, two digit, or three digit number by the preceding questions, then the children can estimate within the halves of the interval. For example, if the answer is more than 10 but less than 100, is it more than 50 or less than 50?

As your students improve in their ability to handle larger numbers and more complex forms of the algorithms, change the problems used in the activities to reflect that growth. These activities can also be readily altered to include decimals or integers for the more advanced students.

Activity eight

This activity is set up as a game to motivate students to improve their guesses. It requires a student to estimate an answer to within a specified range of the exact answer in order to earn a point.

You should begin this activity with simple problems appropriate to the level of your students. You may wish to use only one operation at first, gradually adding others. As the examples become more complex and the numbers larger, the 5-point range for an acceptable estimate should be increased.

Materials needed

A calculator for every two students.

A clock or watch that indicates seconds

A set of cards on which appropriate expressions have been written.

For example:

Easy ones

4×12	$28 - 4$
$19 + 9$	$23 - 12$

Harder ones

6×53	$229 - 16$
$108 + 94$	$116 - 38$

Instructions

1. Shuffle the cards and place them face down.

2. Draw a card and estimate the answer within the time limit imposed (5 to 10 seconds).

3. Determine the exact answer with the calculator.

4. Compare your estimated answer to the calculated answer. Score one point if your answer is within the specified limit (say 5 either way from the calculated answer).

5. The first player (or team) to score 5 points wins. The allowed time, range of acceptable answers, and winning total may be varied according to the level of your students.

Activity nine

This activity requires that a student use a specific operation (not the one usually used) to determine the exact answer to a problem. This requirement is imposed to force the student to estimate and then use the results of that estimate to predict an answer closer to the actual answer. The stu-

dent who understands the inverse relationship between operations rapidly becomes very adept at this activity.

Materials needed

A calculator for every two students.

A set of twenty to thirty cards on which examples have been written. Use only one operation per set of cards and use only one set of cards per activity. The following are some examples for the four operations:

Subtract

$7 - \square = 3$ $36 - \square = 19$ $123 - 46 = \square$

Add

$3 + \square = 8$ $12 + \square = 35$ $227 + \square = 360$

Multiply

$\square \times 4 = 36$ $7 \times \square = 56$ $\square \times 12 = 720$

Divide

$35 \div \square = 5$ $\square \div 7 = 8$ $128 \div \square = 16$

Three levels of difficulty are shown for each operation in these examples. We suggest that these levels not be mixed initially, unless you wish to use the activity as a review for more able students.

Instructions

1. Shuffle the cards and place them face down.

2. Turn a card face up and estimate the answer to the problem shown.

3. Use the calculator and *only* the operations symbol shown on the card to see if your estimate is the exact answer. For example, if the card shows "$7 - \square = 3$, you may check your response *only* by using the subtraction operation and your calculator. Pressing 7 ⊟ 4 ⊜ would give you the right response.

4. If your answer is correct, you receive 5 points. If your answer is not correct, try again. If you get the right answer on the second try, you get 4 points. For each additional estimate, deduct 1 point. You therefore have 5 tries to earn points for your estimate. If you fail to match the exact answer in 5 tries, you receive 0 points and your opponent takes her or his turn.

5. The first player to score 20 points wins.

Summary

We hope the activities suggested in this article will give you an idea of the types of exercises that may be beneficial to your students. Be sure that you use problems appropriate to their level of understanding. Encourage the children to explore their calculators, to try their own ideas, and to ask questions. ■

The Good Professor Takes His Responsibility As a New Standard Bear-er Quite Seriously.

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STANDARD 9:
GEOMETRY AND SPATIAL SENSE

In grades K–4, the mathematics curriculum should include two- and three-dimensional geometry so that students can—

- ***describe, model, draw, and classify shapes;***
- ***investigate and predict the results of combining, subdividing, and changing shapes;***
- ***develop spatial sense;***
- ***relate geometric ideas to number and measurement ideas;***
- ***recognize and appreciate geometry in their world.***

Focus

Geometry is an important component of the K–4 mathematics curriculum because geometric knowledge, relationships, and insights are useful in everyday situations and are connected to other mathematical topics and school subjects. Geometry helps us represent and describe in an orderly manner the world in which we live. Children are naturally interested in geometry and find it intriguing and motivating; their spatial capabilities frequently exceed their numerical skills, and tapping these strengths can foster an interest in mathematics and improve number understandings and skills.

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions about two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Children who develop a strong sense of spatial relationships and who master the concepts and language of geometry are better prepared to learn number and measurement ideas, as well as other advanced mathematical topics.

In learning geometry, children need to investigate, experiment, and explore with everyday objects and other physical materials. Exercises that ask children to visualize, draw, and compare shapes in various positions will help develop their spatial sense. Although a facility with the language of geometry is important, it should not be the focus of the geometry program but rather should grow naturally from exploration and experience. Explorations can range from simple activities to challenging problem-solving situations that develop useful mathematical thinking skills.

Evidence suggests that the development of geometric ideas progresses through a hierarachy of levels. Students first learn to recognize whole shapes and then to analyze the relevant properties of a shape. Later they can see relationships between shapes and make simple deductions. Curriculum development and instruction must consider this hierarchy because although learning can occur at several levels simultaneously, the learning of more complex concepts and strategies requires a firm foundation of basic skills.

STANDARD 9:

- **use problem-solving approaches to investigate and understand mathematical content;**
- **formulate problems from situations within and outside mathematics;**
- **develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;**
- **verify and interpret results with respect to the original problem situation;**
- **generalize solutions and strategies to new problem situations;**
- **acquire confidence in using mathematics meaningfully.**

Activities for the Geometry and Spatial Sense Standard

1. Each child will explore a variety of geometric shapes and see how they could tessellate them or if they would tessellate on a grid. Then we will share our creations by framing them with popsicle sticks and having an art gallery exhibit. (Stephanie Siebeking)

2. Have students play a "Who am I?" game with geometrical shapes. They are given clues such as, I have 4 sides. Each of my sides are connected, and each is the same length. What am I? Students use these clues to form the shape using students as vertices and a large loop of yarn. (Eileen Tully)

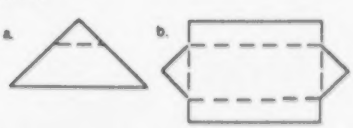
3. I will have the layout of different shapes

   One page just cubes - the kids must decide which ones would make the shape. They would then cut out and put the shapes together to figure out if they are right or not. The children will then get immediate feed back on whether they are right or not. I would have a page for different shapes. Ex.

   (Lisa Potvin)

4. Give each child a picture of a geometric shape, which can be three dimensional, have depth, or any other shape. Have the child construct the shape with blocks, lego's, or other building materials. This activity will enhance spatial awareness. (Shari Wilcox)

5. What shape does each make if folded on the dotted lines?

   (Pauline Payette)

6. Have geoboards for the children and give them rubber bands, have statements/questions on the board for them to answer. "How many different size shapes can you find with three sides? How many different size shapes can you find with four sides? How many different size shapes can you find with five sides?" etc. (Laura West)

7. For the younger grades have them (either at home or at school-walk around) look closely, find and draw as many *recognizable* shapes as they see. This activity will allow them to notice more "geometry" in their environmental daily lives. (Brenda Sloss)

8. Give students pictures of shapes. Then, ask the students to imagine that they are the shape and arrange it a different way. Have them draw those new shapes.

   (Marnea Prestel)

9. How many ways can you arrange 7 toothpicks to form different triangles, parallelograms and rectangles? (Karen Fallin)

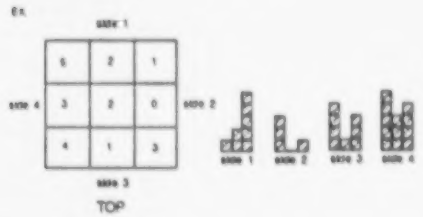
10. Give students pictures containing geometric shapes in them (for example, pictures of buildings, flowers). The student is to look at the picture and write down all the shapes he sees in the picture. Then the student is to tell the characteristics of that shape. (Jennifer Hoelk)

11. Discuss shapes ○ □ △. Go for a shape walk around the school grounds. Identify shapes in the environment. Record findings. (Deena Loan)

12. An activity for geometry and special sense. Each child is given a geometric shape. The children are placed in groups of 4 and are asked to take their

4 shapes and use all of them to create as many figures as possible. Each figure is to be drawn and will be shown to the class during discussion. (Angie Buczek)

13. Give students a 5 part map of a set of blocks, 1 part (view) from each side and from the top. Ask students to build the set.



(Kristen Givens)

14. Each small group is given a shape. They must brainstorm to think of everyday objects which take this shape. They share the results with the class. (Jeanie Mohwa)

15. Children are grouped in 4's. Each group is given 5 yards of string. If each child holds on to string, the children will be able to form geometry shapes (trapezoids, circles, etc...). They also will be able to visualize that they each are a vertex in their geometric figure. (Melissa Baker)

These ideas are from Tech students in the K-8 certification program.

Somehow, Professor Bear Always Knew How to Make Antelopes Behave —

As a Matter of Fact, He Was Known as the Bad Gnus' Bear.

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Simple Symmetries

Grade Level

TOOLS

Paper (some colored)
Paint
Scissors
Crayons
Paste
Old magazines
Cloth or wallpaper scraps

Why

To help young children develop an understanding of bilateral symmetry and a sense of geometric patterns

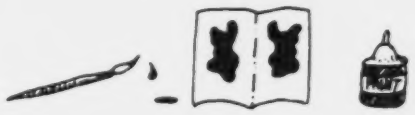
► *Bilateral symmetry shows the same pattern arranged symmetrically on two sides of a center line or axis.* ◄

How

Help your child follow the directions for each design.

Blob Pictures

- ☐ Fold a piece of paper in half.
- ☐ Open it up and drop a blob of paint on one side.
- ☐ Fold it in half again and press.
- ☐ Open it up to see the design that has been made.



Snowflake

- ☐ Start with a square.
- ☐ Fold a square in half.
- ☐ In half again.
- ☐ In half again, on the diagonal.
- ☐ Cut out designs.
- ☐ Open the paper and find a snowflake pattern.

Exerpted from FAMILY MATH. Courtesy of Los Angeles County EQUALS, Sara Munshin, Site Director. (213)922-6680

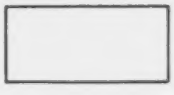
IDEAS

Name _____

FACE OFF

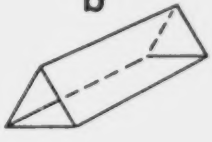
This is one face of a three-dimensional object:

Which of these objects have a face to match it and, how many?

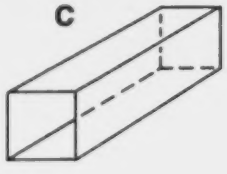


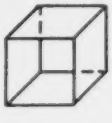
a

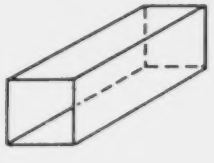
b

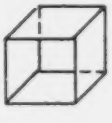


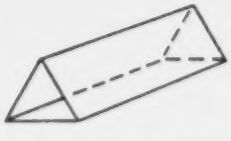
c

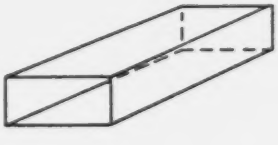












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TRACK CARDS

A Different Activity for Primary Grades

By **John A. Van de Walle**

Do the children in your class *talk* with their friends about mathematical ideas? Are they given the opportunity to plan, to try, to discover, and to create? The development of both the mental capability and the enthusiasm for creative expression and discovery must be nurtured at all age levels and all ability levels.

Track cards are a simple, easily made manipulative. Designs can be created, properties can be investigated, verbalization can be encouraged, and young children can have fun doing good mathematics. The materials described here were originally thought to be most applicable in the special education classroom to encourage verbalization and promote eye-hand skills. Indeed, in terms of spatial perception, eye-hand coordination, and simple problem solving, they have been well received by teachers of special students. However, while exploring the potential of the track cards, I have found them to have considerable value for all children in kindergarten through third grade. Even with an informal approach, the children become quickly engrossed in a variety of good problem-solving strategies. As the children construct roads according to a pattern, or create a unique design, a situation is developed in which they (with or without the teacher) can think, discuss solutions, talk openly, and pursue creative ideas. These aspects alone make the track cards valuable in the primary classroom.

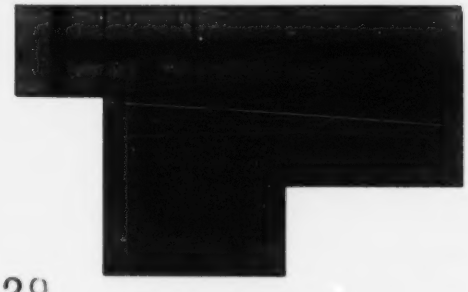
("Track Cards" is the name given to a similar set of materials that are a small portion of the kindergarten program of the Comprehensive School Mathematics Program, a division of CEMREL, Inc. In the CSMP materials, straights and curves are used to discuss the topological concepts of open and closed curves. All discussion and extension of these materials in this article are the author's.)

The Cards

Track cards are made from tagboard or any lightweight cardboard cut into squares. The squares can be of most any size from about eight to twenty centimeters on a side. Larger, sturdier cards are suggested for the very young, but even 10-cm tagboard squares have been used with success in kindergarten. Each card has one of four types of "track"—"straight," "curve," "cross," and "tee"—drawn about 2 cm wide

Fig. 1

Fig. 2



An assistant professor of elementary education and mathematics at Virginia Commonwealth University in Richmond, John Van de Walle teaches content and methods courses at the graduate and undergraduate levels. He also supervises elementary student teachers.

with a dark felt marker. (The cards could also be printed on an offset machine or the tracks could be cut from black paper and pasted on the cards.) A suitable collection for one to four children consists of about sixteen each of the "straight" and "curve" cards, and eight each of the "cross" and "tee" cards. (Fig. 1) The track on each card should meet the edge of the card exactly in the middle. Then any two cards placed edge to edge will form a continuous path. (Fig. 2) These paths (tracks, designs, patterns) are the objects of the various activities with the cards.

An optional layout board can be used to aid in the orientation of the cards. (Fig. 3) Such a board consists simply of a large grid with squares exactly the same size as the cards. A ten-by-ten square grid is adequate for most purposes, but the exact dimensions of the layout board are not important. I have found that a light-colored piece of flannel cloth stretched over a large piece of cardboard with the grid drawn on the flannel helps to keep the cards from sliding around as the children manipulate them.

The Activities

1. Free play

As with all new materials, children should be permitted ample time to explore the cards in their own ways. One child or small groups of children will see quickly how the cards can be laid out to make designs, paths, patterns, and pictures. After only a few minutes of introduction to the cards, the children should be allowed to spend at least one period following their own inclinations with the cards. During this time of exploratory activity, be sure to listen to the chatter of the children and observe the patterns they make with the cards. Build their spontaneous vocabulary and their ideas into your first more structured activity.

2. "Make this pattern."

Make patterns that the children can reproduce with the cards. You can use a sheet of paper of notebook size (21.5 × 28 cm) with a 2-cm square grid. As an introduction to this procedure of following a pattern, kindergarten children benefit from the use of a much larger grid pattern drawn on the board or a large newsprint grid taped on the wall. With the large demonstration grid pattern, the children all work together with the teacher's directions. Once they can reproduce patterns from the large grids, children will be ready to work from smaller versions.

To make patterns, simply take a wide-tip felt marker and trace a heavy path around the grid. As your marker enters each new blank square, decide which of the four track cards you want to represent next and draw the line accordingly. Start with very simple patterns and put only one pattern on a sheet. The patterns in figure 4 are arranged in order of increasing complexity. Make plenty of patterns—it only takes a few minutes for each pattern. A nice collection of twenty patterns could easily be drawn up in a single evening. If patterns are slipped into acetate sheet protectors, they become a durable collection of activity materials.

3. Letters of the alphabet

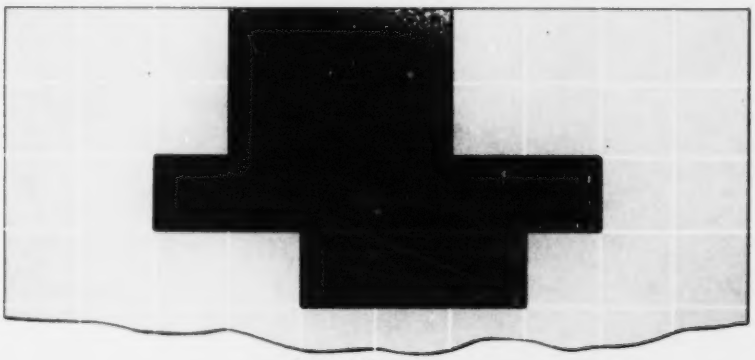
Have children try to make their own initials. With some modifications, all the alphabet can be made using the track cards. The letters with slanted lines (*M*, *N*, *V*, *W*, *Z*) will require that you twist some of the cards (rather than lay them down edge to edge) or that these letters be made more "curvy." In any case, working toward these solutions provides excellent problem-solving and verbal experiences. Using the cards to form letters can be a fun way to discuss the shapes of newly learned letters in the kindergarten class.

4. "How many cards?"

A nice follow-up activity is to ask a series of "how-many" questions as children complete a pattern with cards. Such questions are especially helpful to children who are still developing number concepts and counting skills. The following are only suggestions of the type of questions that can be posed at this time: How many crosses (or tees, straights, curves) in your pattern? How many dead ends? How many cards did you use? How many closed loops do you have?

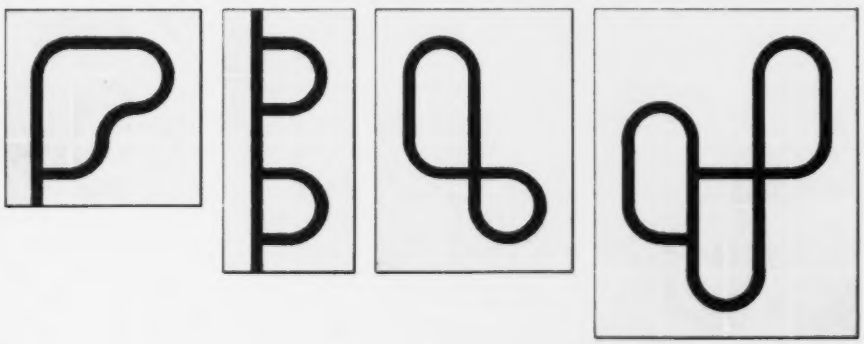
In a similar manner, questions asking for comparisons can be posed and are very important to the

Fig. 3



Layout Board

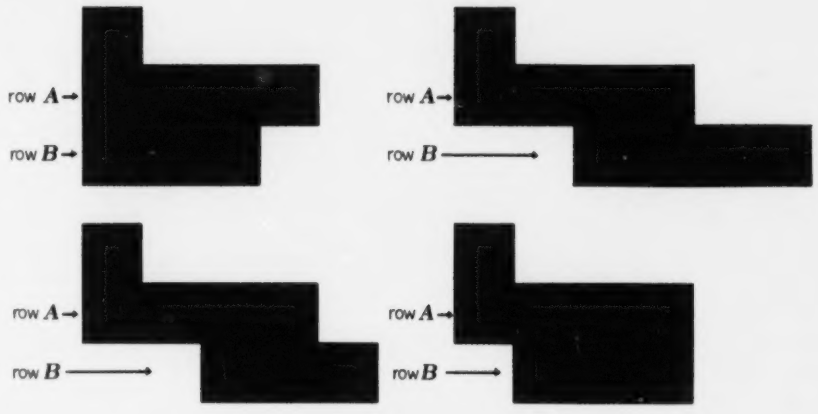
Fig. 4



development of numeric concepts: Which kind of card was used most (or least)? Are there more cards in this row or this one? Which loop do you think is larger and why?

(The question of row length may provide interesting insights into the child's ability to conserve length or number as discussed by Piaget. Consider rows *A* and *B* in each of the drawings in figure 5. For many young children the answer to the question of which row is longer may not be the one we would expect. The track cards provide several opportunities to conduct conservation task experiments informally.)

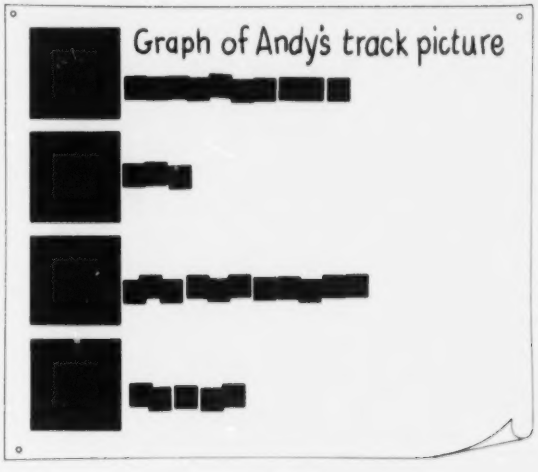
Fig. 5



5. Charts and graphs

The how-many and comparison questions give rise very naturally to simple charts and graphs that children can make. As a start, help children construct a simple bar graph showing the number of each type of card used in their designs or track pictures. A 5-cm square of construction paper can be pasted in the appropriate row for each card in the design. Or, cubical blocks can be stacked up—making a tower of blocks for each card type, using one block per card. Some children will be able to make graphs easily. Children who have trouble making graphs may be helped by having a square of paper (or block) placed on each card of the design. Then, as they pick up each square of paper or block, they paste or put it on the graph. (Fig. 6)

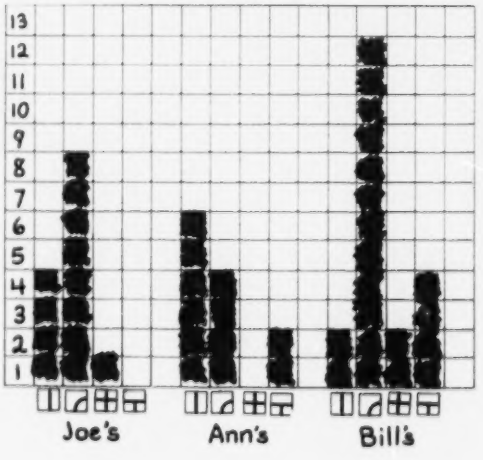
Fig. 6



After children have had some experience with simple graphs, they can make and compare charts or graphs of several different designs. With more experience, children may be able to simply color in squares on grid paper, instead of cutting and pasting squares of paper. The charts and graphs in figure 7 are two different ways to record data for the purpose of organization, comparison, and observation.

Be sure to ask the how-many and comparison questions while you are discussing the graphs and charts. In one kindergarten class, the how-many questions were asked after a simple graph had been built with cubes. Interestingly, the children referred to the graph instead of the track picture for their answers. The superiority of

Fig. 7



	Joe's	Ann's	Bill's
■	4	6	2
■	8	4	12
■	0	2	4
■	1	0	2
Loops	2	1	3
Dead Ends	0	2	1
Total Cards	13	12	20

structured data and its use had been made evident, even to five-year-old children.

The representation of ideas is an extremely important aspect of mathematics that is too often overlooked in the primary grades. Young children find these track-card activities fun and exciting. Older children who have not been provided with such early experiences often struggle needlessly through text exercises involving graphical interpretations. The activities described here give children the opportunity to display work and create organized data from their own experiences.

6. "Make one like this."

All the activities mentioned in this article are suitable for children in kindergarten through second grade. This activity and those that follow may be slightly harder, but they can still be tried in grades one and two.

As a variation of the "make this pattern" activity, use patterns where no grid is marked and no indication of the individual cards is shown. (Fig. 8) To make patterns like this, place a grid sheet under a blank sheet of paper so that the grid shows through. Then draw the design as in Make This Pattern. When children are working from patterns without grids, it is important that you not be concerned with exact duplications of these designs. Many of the children's designs will be somewhat different from what you expected, but you may wish to say such things as, "This design has two loops—what about yours?" or "It looks like you're going to need a tee card pretty soon." In this way, the essentials of the design (topological properties) are emphasized. Two children may make two different designs from the same basic pattern. When this happens, an interesting discussion can develop around how the two designs are alike and how they are different, and why they are both like the original pattern.

7. Create and record

In activities 2 and 6, the children lay the cards out according to a plan or pattern. There is considerable value in reversing this procedure. That is, when children have created a design on their own with the cards, they should have the opportunity to make a picture of their work. This can be done on either grid paper or plain paper (each requires different skills) with crayons or felt markers. Children also can be encouraged to write a few observations about their patterns to go with their drawings. Their observations may include such comments as "my pattern has a loop inside of another loop," or "this pattern has two sides the same, you can fold it in two equal halves." The observations will range from the trivial to the sophisticated and will often be very nonmathematical. (Can we really complain if we have included a bit of language and writing practice in a mathematics activity?) Encourage children to make a little booklet of their designs and observations, or have a group of children display their ideas on the bulletin board.

8. Investigating symmetry

Track cards provide an interesting way to develop the concepts of symmetry. Start with the idea of line symmetry. With track cards, lay out a simple pattern that has a definite line down one side. It will be easier to talk about this line if the layout board is used. (Fig. 9a) Explain that you have made one half of the design and the children are to make the other half so that it looks like your half but is on the other side of the line. It may be necessary to discuss symmetry or mirror image, or even to complete one design to convey to the children what is to be done. Show them other examples of symmetry, such as leaves, butterflies, or folded paint blobs.

Another way of introducing line symmetry is to draw the first half of a design on grid paper, indicating the line of symmetry. Using this pattern, the child makes the pattern with the cards and then completes the design, creating a mirror image. If a mirror is placed on the line of symmetry in the pattern, the result should look like the completed symmetrical design. (Fig. 9b)

Once the concept of line symmetry has been established with the track cards, a number of variations provide interesting and challenging follow-ups. Activities like the following can be used.

(*a*) What happens if the line of symmetry does not touch the design? (Fig. 10)

(*b*) Try to construct a figure *on your own* that has a line of symmetry or "center line." Copy the design you make on a piece of grid paper and use a mirror to check the symmetry.

(*c*) Can you make a design that has two lines of symmetry, one vertical and one horizontal? (Hint: Draw the two lines of symmetry and start with a

Fig. 8

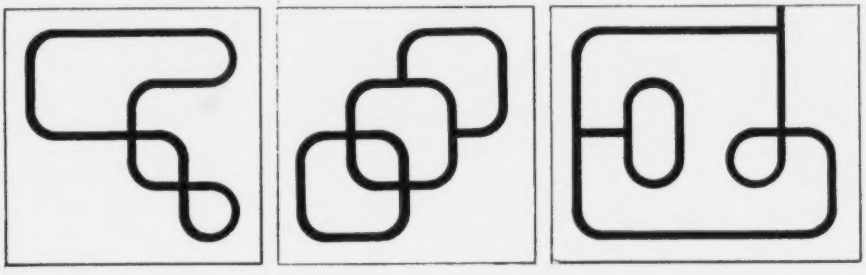
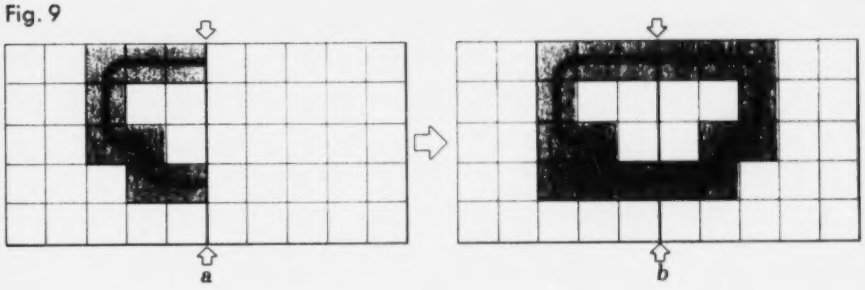


Fig. 9



design in just one corner. Now work from there.)

(*d*) Make a small design. Make its mirror image. Now make a mirror image of the mirror image. What happens?

9. Stretch a design

Another interesting activity is to distort a design in one or two directions—that is, "stretch it" up and down or sideways, or both. The number of loops and intersections does not change when the design is stretched. (These are known as topological properties of a curve.)

It is best to start with the unstretched design on a pattern card so that it can later be compared with the stretched version. (Fig. 11) It will require a bit of teacher assistance to convey to children the idea of stretching a design, but once a child has learned to distort a picture, many open-ended activities and opportunities for creativity will be provided.

Of course, there are no right answers to this activity. Your discussion should include such questions as, How are these still alike? How did you change it? What is different?

10. Verbal problems

One very valuable aspect of the track-card activities is that they provide a setting in which kids tend to be verbal. Even young children can easily talk about the designs they are making and the problems they are working with. Within this free and informal context, you can introduce verbal problems, either written or oral. The purpose of the problem may be to capitalize on a certain concept (small-number counting, for example) or to develop verbal communication or listening skills. In any event, the track cards provide a chance to design real word problems in an easy, nonthreatening atmosphere. The following are just a sample of the problem-solving tasks you can try:

(*a*) Make a track with three loops.

(*b*) If you were building a road from point *A* to point *B*, what is the smallest number of track cards you would need? What is the smallest number of curves you could use? Could you do it with all curves? If so, what is the minimum number of curves?

(*c*) Make a track so that it has two loops. One of the loops should go around this doll and the other around this toy car but the loops should be connected. See if you can do it so that both loops have the same size (or shape or number of cards).

(*d*) Today you are a road engineer, but the people you work for have only given you four curves and five straights. Can you build a loop road? (This one is impossible. Four straights is much easier, but think of the dialogue that this might provoke.)

(*e*) Can you build a path that has two crosses and two loops? What is the fewest number of cards you could use to do it?

(*f*) Try to build a track design that looks like a school bus (or a dog or a house).

(*g*) Make a path that is all connected but has one loop inside of the other, and two ways to get from the outside loop to the inside loop.

(*h*) Return to the charts and graphs discussed in 5. Use these in reverse to create problems. For example, road-builder McCoy wants a road built. He gave us this chart to go by. We must build the road the way he wants it, and the chart tells us what the road must have. Can you help him?

Fig. 10

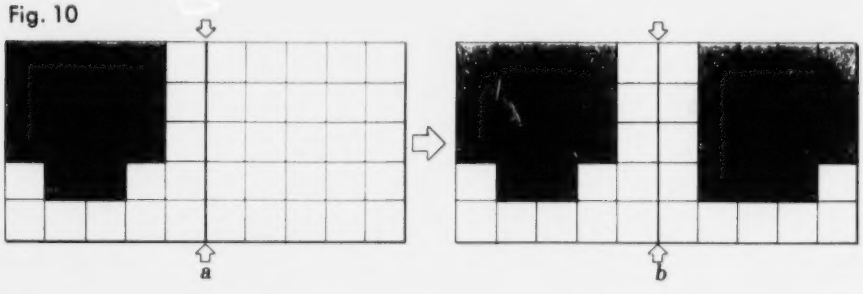
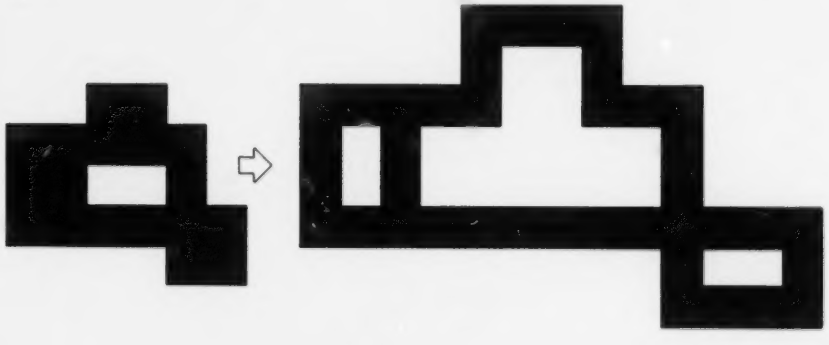


Fig. 11



Summary

The activities presented in this article are intended to be only the beginnings of ideas rather than rigid prescriptions. Teachers and children alike will certainly invent new activities and create variations of other ones.

But first—make some cards! Just as an experiment to get started, cut up one or two sheets of tagboard and make a personal set. *Try* these activities *yourself*, at home or in the teachers' lounge. Then begin to think how your students could benefit from these activities.

It would be interesting to make a list of all the concepts and skills (including art and language areas) normally taught to kindergarten, first- and second-grade children that can be found in the activities described here. Think about this as you wonder how you can fit track cards in with all your other lessons. Perhaps they don't just fit in—perhaps they overlap.

Children (and teachers) who do not (or do not try to) think beyond "getting the answer" are not getting out of the world of mathematics those skills that are most valuable in everyday life. Are the children in your class given the opportunity to plan, discover, or create mathematical ideas? Perhaps track cards will be a beginning.□

Spatial Relationships and Young Children

By **Joseph A. Baust**

"Jane, would you please look under the table next to the wall for my crayon and bring it to me?" To young children, this sentences communicates quite an intricate set of directions that may ultimately help them to find an object in a chaotic environment. Being able to know the position of someone or something in the adult world and being able to discriminate or even crudely measure where these things can be found, is where spatial words and mathematics play a crucial role.

Adults and teachers must recognize that the typical examples of spatial words—*over, under, around, through,* and so on—help children see things in relation to other things. "Johnny, go to the window and look below the windowsill for my pen" relates information that tells the child *where* the pen can be found. If children are to understand this communication, if they are to be successful at making order from a jangle of items and from the mere stated words, then they must have had positive experiences with spatial words.

Getting Acquainted with Spatial Words

Children entering kindergarten may or may not have had experiences in their homes with giving and taking directions. Thus teachers must identify children who have not had the opportunity to listen, comprehend, and act out some directions, and provide such experiences in short but entertaining ways. Even those children who have had appropriate experiences need further reinforcement to provide a commonality of meanings. The following are some examples of fun-filled activities:

Get *in* the circle and act like a tiger.

Go *under* that table (teacher pointing) and make a sound like a car.

Pretend you are a merry-go-round and show how it would go *around.*

It is most important that even initial experiences with directions containing spatial words be simple; they must not have many components. For the young child, the more there is to remember, the greater the chance for failure. Exercises or activities like the ones just mentioned provide simple experiences with taking directions and a common ground for the understanding of simple spatial words, and thus they limit or eliminate opportunities for childrens' failures.

Speaking the Same Language

A barrier to teaching spatial words stems from childrens' confusion about words and their meanings. This is a result of (1) loosely used terminology, (2) incorrectly used words, (3) vague directions, and (4) differences in the children's and teacher's experiences. Teachers must be aware of possible differences between a child's conception of a spatial word and their own. Differences in meanings may be due to a teacher's background and experiences, thus necessitating a careful look at the cultural and other uses of words, as they are used by the children versus as they are used by the teacher.

Often young children may express or act out a word based on what they have heard or seen others do. "Go over to John's house," is markedly different from "Go over to John's table." The child with a limited experience base, however, may respond to either of these directions in a way opposite to what the teacher may intend.

Sometimes, as a matter of convenience or laziness, adults will substitute a word in relating a spatial idea. A phrase that is frequently abused is "Go *over there.*" To the child, the word *over,* as used in this example, may be a source of discrepancy for the child and the word *there* might be the same as anywhere, creating quite a confusing predicament for the young child.

Parents often give children directions like, "It is in the drawer *next to* the window." This statement takes a great deal of imagination on the child's part because many times the drawer may be beneath the window by a considerable distance, or may be one of several drawers in close proximity to the structure. Such a direction presents

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the child with a hard-to-interpret situation based on what is for the child vague instructions. Later, the same parent may show the child, after the child has failed to retrieve the item he or she had asked for, the drawer in question. The drawer may not be even remotely close to the window. This example illustrates two things: (1) From the child's point of view—it is difficult and may be impossible to take directions and be successful. (2) From the adult's point of view—what he or she thinks a word means may not be what the child thinks the word means.

Sorting Spatial Words

It is wise to classify spatial words according to how closely they are related to one another. For example, the following are some common words or expressions and their opposites in meaning:

over	under
above	beneath (underneath)
on	off
on to	off of
top	bottom
on top of	beneath, underneath
high	low
upside down	right-side-up
inside	outside
within	without
in	out
into	out of
before	after
in front of	in back of (behind)
left	right

Words or expressions may also be grouped by teacher needs:

here	there
to	from
towards	away
near	far
closer	farther
close to	far from
next to	among
beside	through
alongside	side-ways
middle	inside out
halfway	
back and forth	

Activities for spatial words should be started on a usage basis. The words should be those most often used by the children in the individual classrooms. A small group of words may be selected to permit a simplistic and effective nucleus of words that children can and will use orally.

Starting with words and their opposites may not be wise because of the difficulty that some children have in seeing inverse relationships or operations. Confusion may result if words such as *over* and *under* are introduced simultaneously. It is important that each child feel comfortable with each word without the burden of having to relate them to another word at the very beginning.

Planning for Teaching Spatial Relations

Once the words to be taught have been selected, the teacher must see that each child—

(a) sees the word in action,
(b) speaks the word with the action,
(c) has an opportunity to do what the words says and say the word,
(d) shares with other children.

A program using seeing, saying, doing, and sharing might be implemented with a "spatial day."

Spatial day

The teacher selects one word. Suppose the word chosen is *under*. Have the word written on the board in big letters, or have it written on a large mobile, or have it written on a paper on every desk. As children arrive, have all of them go *under* a string stretched out across the entrance. The teacher could question the children about what they had to do to walk in the door of the classroom.

All other activities for the day would include the use of the word, *under*, and involve physical tasks associated with the term. For example:

Language arts

Have a story time, with the children sitting *under* a large table in the classroom. Read a book to the children that uses the word *under*. Have the children raise their hands every time the word is used. A book for this purpose could be *I Wonder What's Under*. (Lund, Doris Herold. *I Wonder What's Under*, ill. by Janet McCaffery. New York: Parents Magazine Press, 1970.)

Physical education

Place a group of items—tables, chairs, and other such things—in an obstacle course arrangement. Have the children go through the course. Every time they go under something, they must say "under." The idea could be further reinforced by having the word *under* appearing on the under side of a chair or table that the child must go under.

It is most important that young children are physically active, creatively active, and orally active. Activities like the ones just described provide many opportunities for children to use spatial words effectively and without hesitation. This is what Sylvia Ashton Warner might allude to as an "organic vocabulary"—words of a spatial nature that children can comprehend because they know what the words "feel like."

Achieving Positive Effects

Spatial words are mathematical words that teach position and order. Each word should be introduced in an active rather than a passive mode. Young children should learn spatial words by *doing*, not watching. Through active involvement in learning children will be far more capable of using words such as *under* with greater meaning.

Teachers should provide a positive model for using spatial words during the school day. When speaking or giving directions to children, teachers must be conscious of the meaning intended and, through usage, leave little doubt of this meaning. Have children give directions and let the teacher carry these out. The teacher's modeling is the most important part of providing an environment conducive to learning spatial words.

Much care must be taken to avoid words that are similar but have disparate meanings. Care must be taken to assure a commonality of meaning in all communications involving spatial words.

If the methods that teachers use when they are teaching spatial relations are a result of being mindful of the intricacies of understanding spatial words, children will have positive initial experiences within a most important mathematical area. ■

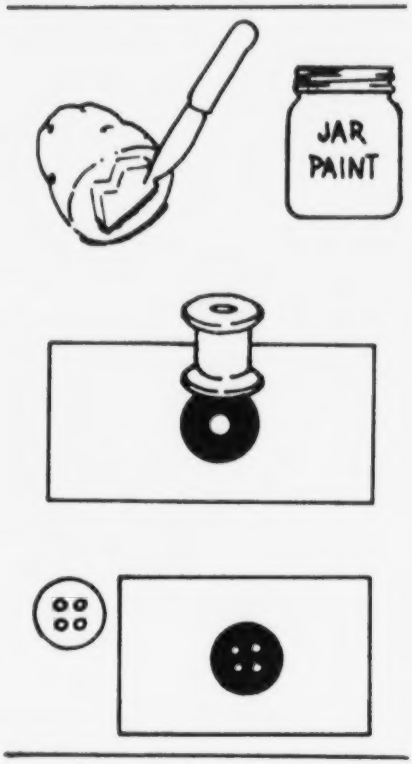
Geometry in the Primary Grades: Exploring Geometric Ideas in the Primary Grades

By **Helene Silverman**

Many of the planned experiences for exploration of materials in the primary grades offer opportunities for the introduction of geometric concepts. After the initial exploratory activities, the teacher can guide the children into some of the more structured experiences that are described in the paragraphs that follow.

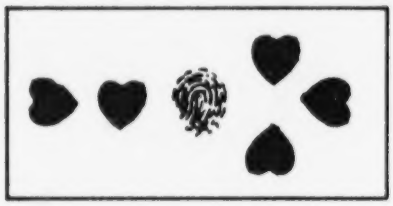
Describing Motions

Transferring the shape of an object onto paper by printing can be very exciting for children. Many children are introduced to printing through commercially prepared stamp pads and stamps, but children can also print using spools, cut potatoes, carrots, cheese, nutshells, buttons, wood bits, and cardboard or oaktag pasted to corrugated board. Water-based ink or tempera paint can be brushed on any of these objects, and the pattern can be "stamped" on paper.



A single figure can be stamped repeatedly along a strip of paper. Children can then describe the various ways in which they have moved the stamp and compare the resulting imprints.

The child's fingerprint can be used as a reference point from which all other motions can be described. Some typical comments from the children might include the following: This shape was moved over and up. This shape was turned around. This shape was turned around and moved away. Most children will be able to perform the motions, but they will need help from the teacher in getting some of the language—up and down, left and right, turned, around, and so on.



Additional experiences can be provided through the use of templates, which can be either purchased or made out of oaktag or styrofoam trays. For the latter, the child reproduces the design by tracing out the hollow with a pencil or crayon. As children reproduce the design in several places on their papers, the movement of the resulting shapes from one place to another can be described. Children can also determine whether any "new"

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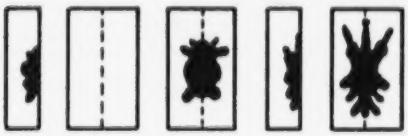
kinds of imprints can be made by turning the figure over.

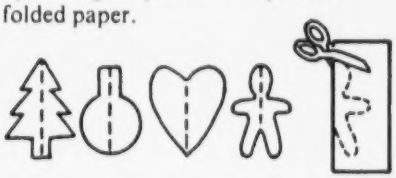
Objects that children have found can also be used to demonstrate various motions as the children observe the differences in position resulting in moving objects left, right, up, down, turning them around, and flipping them over. Children can also be encouraged to record some of the possibilities.

Observing Bilateral Symmetry

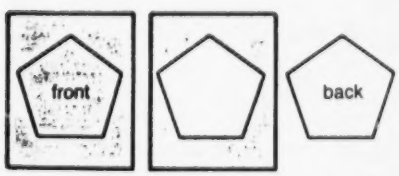
Blot prints can be made by folding a piece of paper, opening the paper, applying a blot of water-based ink or tempera paint at the fold, refolding the paper, and pressing the folded paper. The children can compare matching parts on each side of the fold.



Children may enjoy folding things that can be shown to have matching halves. Some children may also be able to create their own symmetrical figures by cutting shapes out of a piece of folded paper.

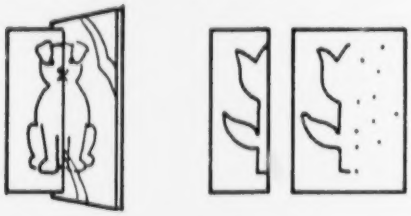


If shapes have been cut out of paper or oaktag, some children may discover that the shape that has been cut out can be fit back into the hole in two ways—in the position in which it was cut and in the position obtained by flipping the piece over.



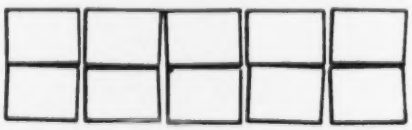
Some children may be challenged to make cutout figures that can fit into their holes in more than two ways. The activity can be a focal point for a discussion of lines of symmetry. Extensions of the cutouts can be made by repeated folding of paper before cutting and the various symmetries that result can be explored.

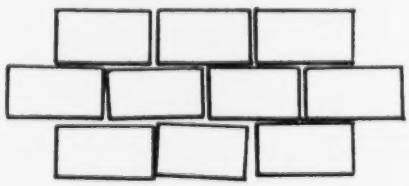
"Hidden pictures" can be found by placing mirrors along half-drawn pictures or by pricking through key points in a design to complete figures.



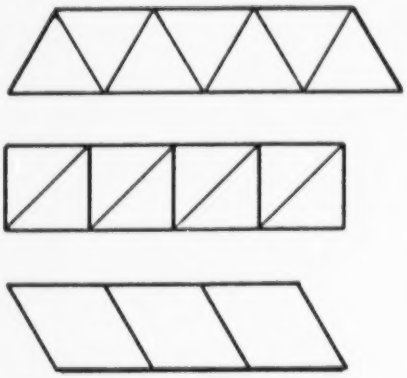
Combining Shapes

Young children generally enjoy using packets of shapes to make pictures and designs. Fitting identical shapes together to make an overall pattern is an introduction to tessellations. Square and rectangular shapes are easiest to begin with, as children put shapes together and extend the pattern left and right and up and down the page.



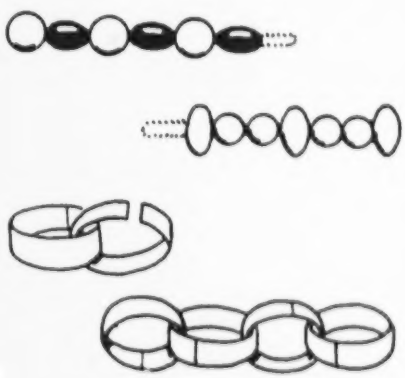


Later the children can learn to rotate shapes in order to fit the shapes together as they work with equilateral and right triangles and with parallelograms or "kites."



Some children will also discover that there are shapes that do not tessellate well alone but do combine well with other shapes to form pleasing overall patterns.

Rows of beads strung on a pipecleaner or a shoelace are fun for children to make and can be used to have the children discover repeating patterns. Working with paper chains is another way to provide additional opportunities for children to describe relative position among the parts of a repeated design as well as within the overall pattern.



Other ways for children to explore motions, symmetries, and combinations of shapes abound. Experiences with moving their own bodies, with observation walks, and with examination of patterns on wallpaper (see "Wallpaper Capers" in this same issue), fabric, wrapping paper, and needlepaper can serve as additional sources of geometric experiences for children. Such experiences will help to make children aware of the geometric relationships that surround them and will provide a good foundation for any later study of formal geometry.□

Let's Do It

Independent Mathematics Center for Everyone

By **Mary Montgomery Lindquist**
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Orchard Ridge Elementary School
Madison, Wisconsin

The idea of a mathematics center is very appealing to many teachers. At centers, you can have the sort of mathematics activity that is not done usually by children at their desks or by the whole class at one time. Such activities can be fun as well as instructive and thought-provoking. Unfortunately, many centers take too much time and effort in preparation and can also require too much teacher direction. We have suggested several sample activities, appropriate for grades one through three, that involve minimal teacher preparation, materials, and direction.

Activities such as these we are suggesting can be set up at a mathematics center for a week or more at a time. Each day a few children can visit the center, whenever time allows. You might want to write a schedule on the board that tells on which day each child can visit the center and then let the children find time that day. Some activities might require pairs of children, and they can be scheduled accordingly.

One important aspect of a successful center is that the children record in some way what they have done. This helps you see what sorts of successes or problems the children had; it also discourages children from fooling around at the center and not accomplishing anything. Well-designed centers can encourage independence and responsibility in children, as well as developing or reviewing the chosen concepts.

A mathematics center of the sort we are describing need not take up a lot of space. It can be off in one corner or

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perhaps even out in the hall where it won't be too distracting.

You will want to save the duplicating masters and other materials at a center for use again next year or you might share them with other teachers at a workshop or meeting. Once you have developed a good center, get as much use out of it as possible.

A Weight Center

This center is intended to be used during early first grade. The idea of using a balance beam to order the weights of two objects is appropriate for this age level and the form of recording the answers is designed for nonreaders. You will need to give some oral direction to the children so that they know what the center is about and what to do at it. One way to do this is to introduce the center to the whole class. Another way is to explain it to the first child and have that child explain it to the next child who comes to the center. You might want to list the children's names in the order that they should visit the center.

Main idea(s)

Deciding which of two objects is heavier, using a balance beam

Materials

One balance beam
Six objects to be weighed
Masking tape and marker
Duplicating master

Preparation

1. Choose six objects of different weights, including some whose size is misleading (such as a sponge, which is large but light). Label the objects with masking tape and marker. (Letters used in the example are *a*, *g*, *k*, *s*, *m*, *e*).

2. Prepare a recording sheet (fig. 1) using the letters you have used to label the objects. Make a copy for each child.

Tell the group, or the first child, to (a) get the two objects named in the first box on the recording sheet, (b) put them on the balance beam, and (c) circle the letter of the one that is heavier. Explain that if the objects are the same weight, both letters must be circled.

Fig. 1

Name _____

Circle the heavier one.

a g	a k	a s
a m	a e	g k
g s	g m	g e
k s	k m	k e
s m	s e	m e

To challenge some children, you can make up a sheet that names the objects in threes and ask children to circle the heaviest of each three. Since three objects cannot be compared on the beam at the same time, the children must use logic to help them decide which is heaviest.

A Counting Center

This particular center is appropriate for children who would benefit from counting objects numbering from 20 to 100. The activity also encourages children to estimate quantities of this size. Children work in pairs. Each child guesses how many objects are in a container; then, the children count to see whose guess is closer. Even if children do not have an algorithm for finding the difference between two two-digit numbers, they can often figure out whose guess is closer.

If you want to encourage "good guessing," set up the containers so that guessing one may help guess the next. You might fill jars of various sizes with one type of filler (such as marbles). Or you could use containers of the same size and fill them with different objects (one with marbles, one with macaroni, one with cubes, and so on.

Main idea(s)

Guessing and counting objects, numbers less than one hundred

Materials

Seven containers
Filler for seven containers
A duplicating master
Tape and marker

Preparation

1. Label the containers *A* through *G*. Fill the containers.

2. Make a duplicating master. The example in figure 2 encourages the children to count in groups of ten.

If containers or fillers are too difficult to obtain, you could laminate pictures that show numerous objects. The children can use a crayon to help them keep track. If you are stressing place value, you could have the children circle groups of ten to assist in the counting.

Fig. 2

Each of you guess how many things in A. Then count the objects. Circle the guess that is closer. Repeat for B - G.

	____'s guess	____'s guess	____'s guess	Count
A				tens ones
B				tens ones
C				tens ones
D				tens ones
E				tens ones
F				tens ones
G				tens ones

Winner: The player with the most guesses circled.

A Puzzle Center

This center is designed for children who cannot follow printed directions. They can work in pairs or alone, and simply record which puzzles they can solve by checking them on a recording sheet you have made. You will undoubtedly think of variations on the basic puzzle idea and will want to add your own to the center.

Main idea(s)

Practicing eye-hand coordination
Using logic to solve a puzzle

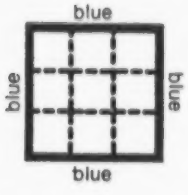
Materials

Wide-tip blue marker
Tagboard or other stiff paper
Two duplicating masters

Preparation

1. Make an 18-cm-by-18-cm square shape. Color the sides of the square with a blue felt-tip marker; then cut the square shape into nine square shapes, each 6 cm by 6 cm, as in figure 3. Make one such set of square shapes for each of the children who will be using the center at any one time.

Fig. 3

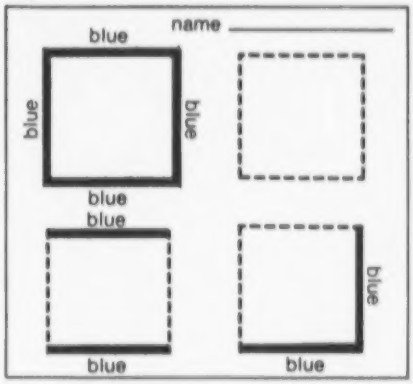


2. Make a worksheet showing a square of the same dimensions (18 cm by 18 cm) and the cut lines, but do not mark the sides of the blue square. Run off several copies for the center. Put one set of square shapes with each worksheet.

3. Make copies of a recording sheet like the one in figure 4.

You will need to explain to the children as a class, or at least to the first group of children using the center, what they are to do. They are to arrange the small square shapes to make a large square shape the same size as the one on the worksheet. They are to

Fig. 4



do so in each of the four ways shown on the recording sheet. First, they arrange the small square shapes so that the large square shape is blue on all sides. Second, they arrange them so that the large square shape is not blue on any of its sides. Third, they arrange them so that two opposite sides of the large square shape are blue and the other two sides are not blue. And fourth, they arrange them so that the two sides next to each other on the large square shape are blue and the other two sides are not blue. As a child solves each puzzle, he or she checks on the recording sheet the one completed, until all four are done. If a child is stuck, another child at the center can help.

A Length Center

At this center for first- or second-graders, the children measure in centimeters to make a picture of a rocket. Each child follows simple directions in making the parts of a rocket the lengths indicated. Thus, the children need to be familiar with measuring in centimeters and with making something a given length. The rocket pictured in figure 5 is an Apollo rocket scaled so that 1/2 centimeter represents a meter. Have children work individually on their rockets, unless there are some children who would benefit more from working in pairs.

Main idea(s)

Creating an object of a given number of centimeters in length

Materials

Large sheets of construction paper
One meter stick
One pencil
Crayons
A card for directions

Preparation

1. Prepare a direction card as shown in figure 5.

2. Cut a strip of paper (10 cm by 60 cm) for each child.

Fig. 5

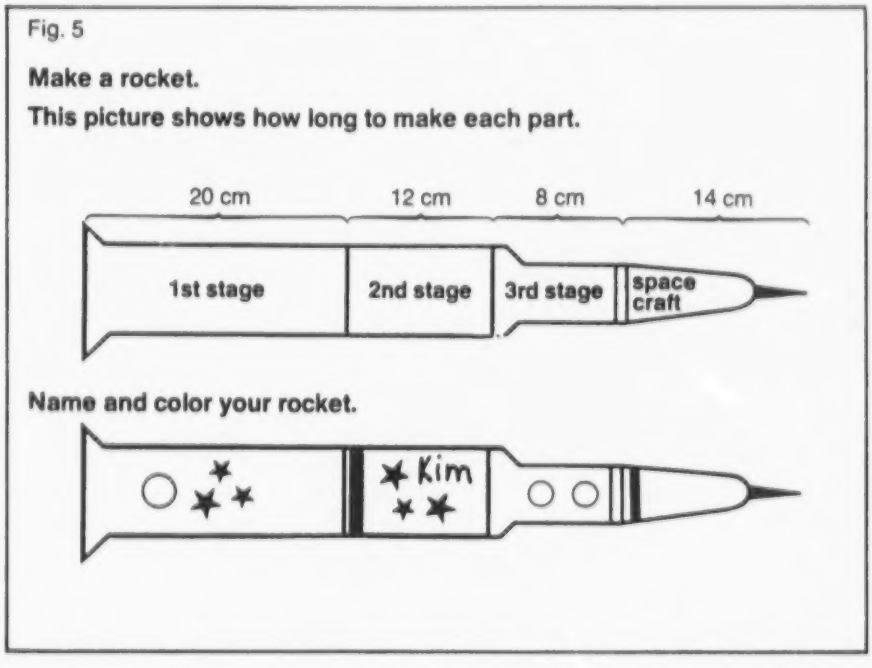
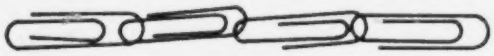


Fig. 6

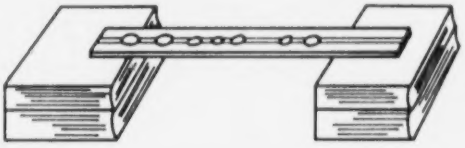
CARD C
Clip Connecting

Connect paper clips in a chain.



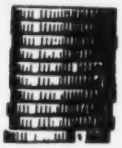
CARD B
Bean Balancing

Balance beans on a meter stick.

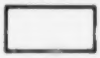


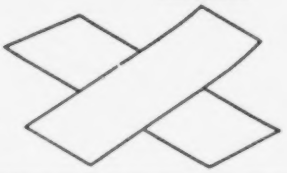
CARD P
Penny Piling

Pile pennies in one pile.



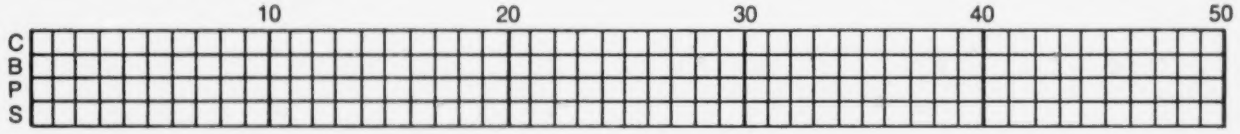
CARD S
Sheet Stacking

Stack sheets of paper, first this way
then this way 



WHAT I CAN DO IN ONE MINUTE!

Name _____



1. Which did I do the most of in one minute? C B P S
2. Which did I do the fewest of in one minute? C B P S
3. Which did I do over 30 of in one minute? C B P S
4. Which was easiest for me and why? _____
5. Which was hardest for me and why? _____

When children have finished their rockets you may wish to display them. Some children may enjoy finding out about other types of rockets and making pictures of them.

A Graphing Center

This center is suitable for any children who can read and follow directions on their own and who are very familiar with the numbers through 100 (probably anywhere from mid-second grade to mid-third grade). The children work in pairs and it may take as much as a half-hour for each pair. You may want to try the four one-minute activities yourself to see about how many of each materials you will need.

Main idea(s)

Making a graph to show the results of several activities
Interpreting a graph

Materials

Paper clips
Pennies (or a suitable substitute)
Stack of paper
Lima beans or macaroni
Meter stick or other board
Books
Direction card
Minute timer or watch with a second hand
Index cards
Duplicating master

Preparation

1. Prepare four index cards and a recording sheet something like those in figure 6.

2. On a large card, print directions something like the following:

Directions

(1) Work with a partner.

(2) One of you be the timer. The other does what each card (C, B, P, and S) says to do for 1 minute.

(3) When you have done what each card says to do, count the objects you got for each. Graph the results.

(4) Switch jobs with your partner.

(5) Answer the questions about your graph.

3. Place all the materials at the center. Make two piles of books and place the meter stick with an end on each pile.

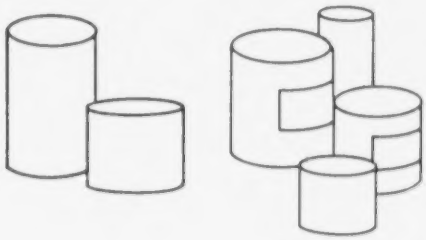
You might want to have extra pieces of graph paper available so that children can extend their graphs if they need to. When all the children have done their graphing, it might be interesting for the class to compare the graphs and discuss which activities were hardest or easiest for everyone.

Fig. 7

CANS

1. Line up the cans, from the lightest to the heaviest.
2. List the cans, from the lightest to the heaviest:

	can	weight
lightest	_____	_____
	_____	_____
	_____	_____
	_____	_____
heaviest	_____	_____



3. Choose two cans, _____ and _____. Which is heavier? _____ How much heavier? _____
4. Choose two other cans, _____ and _____. Which is heavier? _____ How much heavier? _____
5. What cans weigh between 200 g and 400 g? _____
6. How much do the three lightest cans weigh together? _____

An Ordering Center

This center, in which the children order three-digit numbers that are the weights of food cans (or cereal boxes) and solve problems with those numbers, is most appropriate for third-graders because of the size of the numbers and the reading involved. As with the other centers described in this article, however, the activity could be made appropriate for other ages with slight modifications.

At this center, each child takes the ten food containers and puts them in order, from lightest to heaviest, by the weights given on the cans or boxes. After the order is recorded, questions requiring adding, subtracting, and ordering are asked.

Main idea(s)

Ordering three-digit numbers and solving problems involving these numbers

Materials

Ten food cans or cereal boxes with weight given in grams

One duplicating master

Preparation

Prepare a duplicating master like the one in figure 7. Make the questions appropriate for your children.

If children enjoy this activity, you can substitute other cans and the children can repeat the activity. You could simplify the reading level by labeling the cans with letters. Then, problems could be written as follows:

Sum of weights A and B _____

Sum of weights C and D _____

Difference between the weights A and B _____

Difference between the weights C and D _____

We hope these ideas are general enough for you to create your own centers. Remember to keep the centers simple, the directions for activities easy, and to have children record their work in some way. □

NIMble Calculator

Equals Project

The ancient game of NIM meets the ubiquitous CALCULATOR. The only needs are two people and a calculator.

Each version has a starting number, a target number, and a range of numbers (the "take") which you can add (in the "up" versions) or subtract (in the "down" versions). The winner is the first person to reach the target number.

Challenge: Find a strategy for winning. (Some hints are given at the end.)

I. 7 UP: Clear the calculator so it reads 0. Each player on his/her turn adds 1 or 2 into the calculator. The winner is the first person to reach 7. Going over 7 loses.
Start: 0
Take: Add 1 or 2
Target: 7

II. 11 DOWN: Clear the calculator and enter 11. Subtract 1 or 2 on each turn. Winner is the person to reach 0.
Start: 11
Take: Subtract 1 or 2
Target: 0

III. NOW YOU'RE 21: Clear the calculator so it reads 0. Add 1, 2, 3, or 4 on each turn. The person to reach 21 wins.
Start: 0
Take: Add 1-4
Target: 21

IV. TRAVEL DOWN 101: Enter 101 on the calculator. Subtract 1, 2, 3, 4, 5, 6, 7, 8, or 9 from the number in the display. Winner is the person to reach 0.
Start: 101
Take: Subtract 1-9
Target: 0

V. CENTURY: Start at 0 and add 1-9 on each turn until 100 is reached.
Start: 0
Take: Add 1-9
Target: 100

VI. 2001: Enter 2001 in the calculator. Subtract a number from 1 through 99 on each turn. The person to reach 0 wins.
Start: 2001
Take: Subtract 1-99
Target: 0

Search for a strategy: here's a way to begin.
Play one version at least 5 times. Each time the loser chooses whether to play first or second the next round. Notice at which point in the game you know which is going to win.

Next level of analysis:
Consider the simplest case. Explore the possible moves and resulting winners. Look for patterns.

Source: EQUALS Project, Lawrence Hall of Science, UC Berkeley.

Estimation and Mental Computation

NCTM's 1986 yearbook makes a strong case for the importance of estimation in various domains, such as numerosity, computation, and measurement. A recurring theme throughout the book is the interdependency between the process of estimating in a particular domain and the understanding of mathematical concepts within that domain.

The book is divided into five sections. Part 1, "Introduction," provides a framework for the yearbook. Part 2, "Teaching Estimation and Mental Computation," presents specific instructional activities. It contains programs of study for teaching in the primary grades. Part 3, "Varieties of Estimation," discusses the various types of estimation and uses for estimation. Part 4, "Teaching Measurement Estimation," has teaching activities and mathematical topics appropriate for grades K-12. Part 5, "Testing and Research," presents an evaluation of computational estimation and a summary of research on teaching and learning estimation.

The 1986 Yearbook is available from the NCTM Headquarters Office for $16 a copy (prepaid; ISBN #0-87353-226-0). NCTM members get a 20% discount.

Poetry and Mathematics

JONATHAN BING DOES ARITHMETIC

When Jonathan Bing was young, they say,
He slipped his school and ran away,
Sat in the meadow and twiddled his thumbs
And never learnt spelling or grammar or sums.

So now if you tell him, "Add one to two,"
"Explain what you mean," he'll answer you,
"Do you mean **2**-morrow or that's **2** bad?
And what sort of **1** do you want me to add?

For there's **1** that was first when the race was **1**
(For he ran **2** fast for the rest to run).
But if **2** had **1** when the race was through,
I'd say your answer was **1** by **2**."

"Oh Jonathan Bing, you haven't the trick
Of doing a sum in arithmetic."
"Oh give me a chance, just one more try,"
Says Jonathan Bing with a tear in his eye.

"Very well, Jonathan, try once more,
Add up a hundred and seventy-four."
"A hundred and seventy-four," says he,
"Why — that's a great age for a person to be!"

— Beatrice Curtis Brown

IS SIX TIMES ONE A LOT OF FUN?

Is six times one a lot of fun?
Or eight times two?
Perhaps for you.
But five times three
Unhinges me,
While six and seven and eight times eight
Put me in an awful state
And four and six and nine times nine
Make me want to cry and whine
So when I get to twelve times ten
I begin to wonder when
I can take a vacation from multiplication
And go out
And start playing again.

— Karla Kuskin

4x6 5x3 8x8
6x7 6x1 9x9

SHAPES

A square was sitting quietly
Outside his rectangular shack
When a triangle came down—*kerplunk!*—
And struck him in the back.
"I must go to the hospital,"
Cried the wounded square,
So a passing rolling circle
Picked him up and took him there.

-Shel Silverstein

PRIMARY TEACHING IDEAS

Primary

- Barbara Maguire Shute
Chancellor School

1. > <

When teaching Greater Than and Less Than, try this:

Greater Than

Less Than

Since most children can relate to Pac Man, I tell my students that Pac Man is a number gobbler, and of course opens his mouth to the largest number.

16 > 8

2 < 4

---

2. METRIC

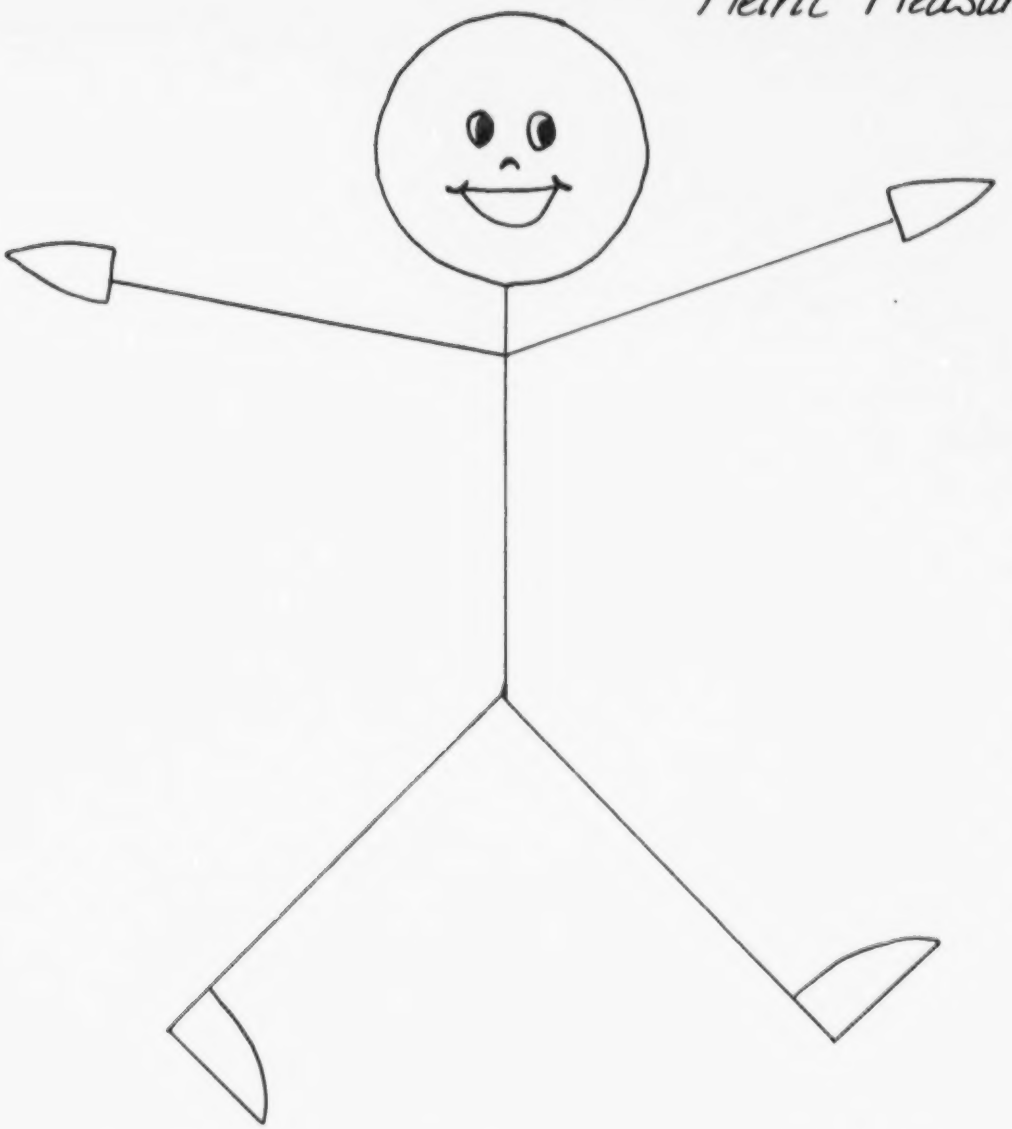
For practise in Metric Measurement these ideas could be helpful. (See the next page) The students can make their own ruler to measure the figure.

Extention Activity: Students may measure the length of their own pencil, crayons, etc. Graph the results and compare.

3.

An Addition game is included, in the next few pages, for Valentine's Day. Two dice are needed. See "Roll and Add" for the directions.

Metric Measurement



width of head ___ centimeters

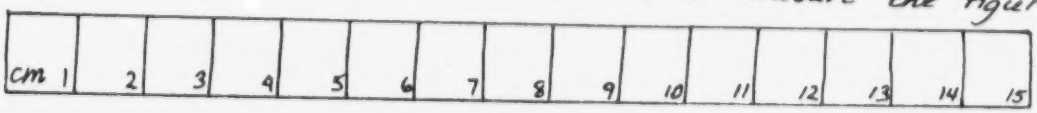
neck ___ centimeters

arm ___ centimeters

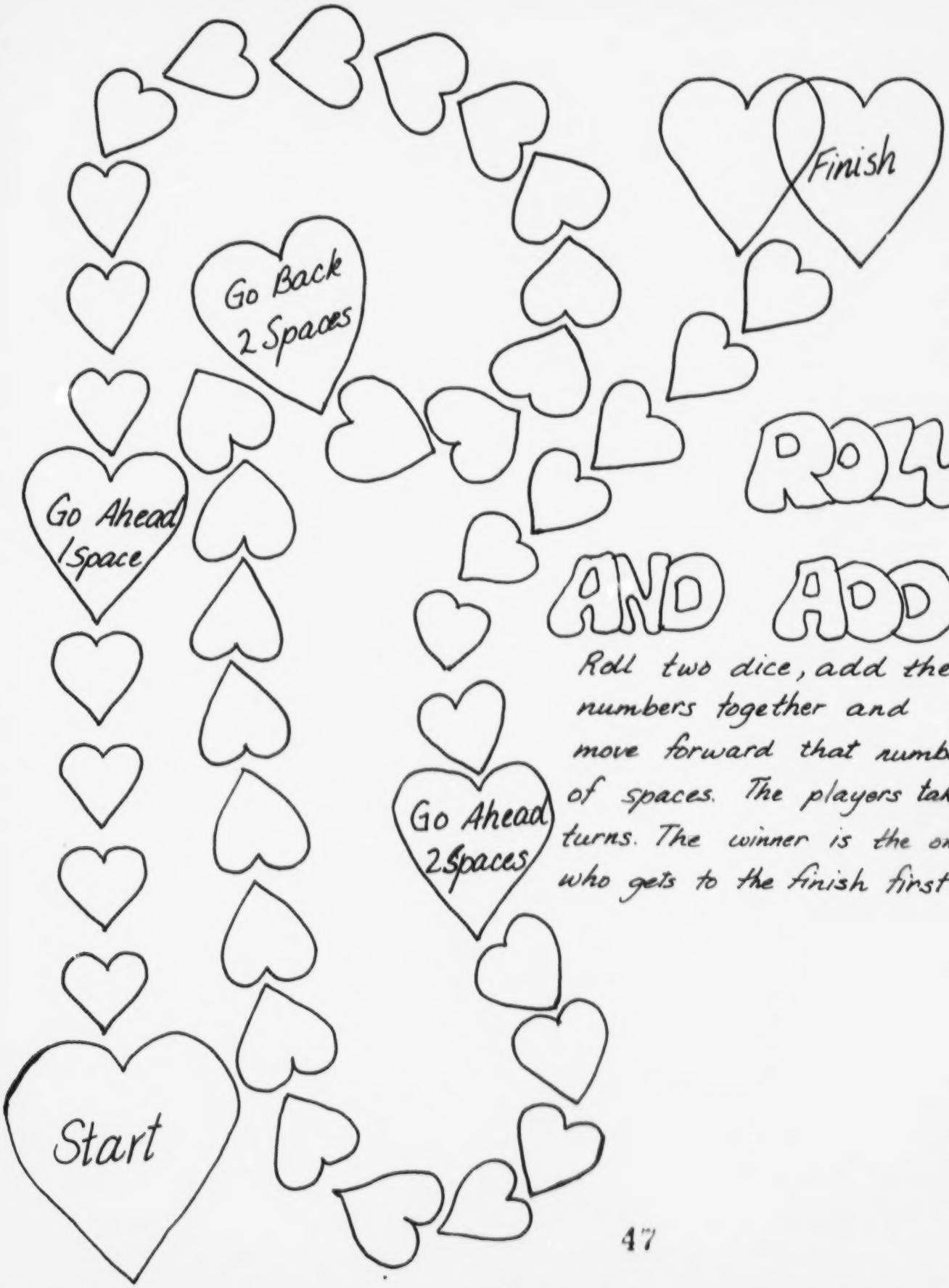
leg ___ centimeters

foot ___ centimeters

Cut out the ruler and use it to measure the figure.



Centimeter Ruler



ROLL AND ADD

Roll two dice, add the numbers together and move forward that number of spaces. The players take turns. The winner is the one who gets to the finish first.

47

Recommendation to the Virginia State Board of Education from the Virginia Council of Teachers of Mathematics

The Virginia Council of Teachers of Mathematics (VCTM) requests that the State Board of Education adopt standardized and competency mathematics tests which permit the use of calculators.

Rationale

Computation is nothing more than changing the name of a number. To illustrate, if we have 7 dozen eggs then it is necessary for educated people to know that we have 7 x 12 eggs. 7 x 12 is the number of eggs. We use computation to change the name 7 x 12 to 84. In the past when we were unable to change the name of a number (calculate) mentally we used a tool—paper and pencil. Some may have called paper and pencil a crutch. Today, in instances when we are unable to calculate mentally, we and our youth should use the most modern tool available to us—a calculator. This is the twentieth century and we are educating youth for the twenty-first century. It is imperative that we permit students to use the most modern and efficient tool available for computation.

Inexpensive calculators can be purchased which not only add, subtract, multiply, and divide integers but perform these operations with common and decimal fractions. The calculators add 2/9 + 5/8 just as easily as they add 6 + 4. The same inexpensive calculators subtract 4 5/8 - 2 3/17 as easily as they subtract 123 - 87.

Permitting students to use calculators on mathematics tests will mean that teachers will no longer feel compelled to use so much instructional time on teaching paper and pencil algorithms. Teachers continue to use valuable instructional time on algorithms because standardized and competency tests still evaluate the students' ability to perform these outdated algorithms.

Instructional time must be spent on number sense and numeration, concepts of number operations, problem solving, applications of mathematics, estimation, alertness to unreasonable results, measurement, geometry, and statistics and probability.

Recent reports from the National Council of Teacher of Mathematics and the National Research Council promote the use of calculators. The inclusion of the calculator in aspects of school mathematics, including classwork, homework and *testing* has been recommended by prestigious groups such as the National Council of Teachers of Mathematics, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematics Education, and the National Council of Supervisors of Mathematics. The State Board of Education in Connecticut has mandated the use of calculators for eighth grade students on the state's mastery test.

The benefits for use of calculators are well documented and supported by research. Even for those who still feel that paper and pencil computation is important, research shows that students who are permitted to use calculators perform as well or better than when using paper and pencil for computation. Research also indicates that students using calculators possess a better attitude towards mathematics and an especially better self-concept in mathematics.

The mathematics curriculum must change. The mathematics curriculum will change only when the curriculum which is tested is changed.

If Mathematics Class Were Managed Like a Well-Run Dining Hall . . .

Cub Students early to Mathematics Class After a hard recess

NCTM Position Statements

Metrication

Measurement is a common daily activity performed in all nations, and a knowledge of its concepts and skills is an important topic for all sectors of society. On an international level, the increased demand for a common language of measurement has been eased by the adoption of the metric system. The teaching of this system of measurement is not the sole responsibility of mathematics teachers, however; all teachers and administrators share the responsibility to prepare students for a world where the metric system of measurement prevails. Yet, many in education, including mathematics education, continue to ignore the teaching and learning of the metric system.

As a part of the mathematics program, metric education provides models for activities involving numeration, decimal concepts, and estimation. It serves as a link that joins measurement concepts and skills to other major topics in the school mathematics curriculum.

Therefore, the National Council of Teachers of Mathematics supports the use of the metric system as an integral part of the mathematics curriculum at all levels of education and recommends that all educators assume responsibility for providing leadership and direction in metric education.

Professional Standards for Selection and Implementation of Instructional Materials

The content, organization, and style of instructional materials, particularly basal textbooks, are among the most important factors influencing outcomes of school mathematics programs. Thus, the process of selecting new textbooks and ancillary materials is a critical step in planning curricula and preparing for instruction. This process includes setting criteria, screening materials, making choices, and implementing the new program. Although teachers and students are vitually concerned with the quality of the instructional materials they must use, the selection and preparation for use of those materials are often constrained by inappropriate selection and implementation processes.

In order to improve curriculum and instruction in elementary and secondary mathematics programs, the National Council of Teachers of Mathematics has adopted the following standards for selecting instructional materials and implementing programs using those materials. The Council urges compliance with these standards by all school districts.

Standard 1:
The Review and Selection Committee

The entire process of textbook selection should be led by teachers and supervisors with expertise and responsibility in mathematics education. Except in most unusual circumstances the recommendations of the committee must be followed.

Membership on the Review and Selection Committee should include, but not necessarily be limited to, classroom teachers representing the grades or courses for which instructional materials are being selected as well as teachers in grades or courses that preceed or follow the target courses; mathematicians or mathematics educators with knowledge of content, curriculum, and pedagogical trends covering the full spectrum of K-12 mathematics and beyond; and school administrators who have overall responsibility for supporting and guiding the implementation of new programs.

Selection of committee members must be made in consultation with a person having expertise and responsibility in mathematics education. Whoever is chosen should be a full working member. Where the talents of a school or district mathematics staff cannot fulfill the roles described above, appropriate outside consultants should be obtained.

Standard 2:
Criteria for Selection of Instructional Materials

Criteria for selection must be defined and made prior to the review process.

The criteria should identify the desired instructional approach as well as the mathematics content. Criteria for selection should be based on program needs which can be identified by:

- evaluation of the present program, including analysis of student achievement, attitudes and learning characteristics, and a survey of teacher opinions;
- occurent research on the teaching and learning of mathematics;
- recommendations of professional organizations; and,
- curriculum guidelines of local, state or provincial, and national governments.

Difficulties in implementation should be considered only if they seem insurmountable. Any adoption of new instructional materials will create some problems of implementation. However, the committee's goal should be the identification of the best possible program, not the easiest to implement.

Copyright dates and formula-determined reading levels are generally inappropriate as selection criteria. Although fashions in content and pedagogy change, a newer (or older) copyright does not assure higher quality. Because of the large technical vocabulary associated with mathematics, results of standard readability formulas should be used with extreme caution.

Standard 3:
The Review of Instructional Materials

The Review and Selection Committee should actively seek materials from a variety of sources and evaluate the materials on the basis of the previously defined criteria.

The process of identifying, collecting, and examining material for potential adoption takes time to be done well. Time for committee members to do their work must be made available by the school system.

The committee should consult the following sources to identify appropriate instructional materials:

- critical reviews in professional journals;
- recommendations of teachers and supervisors in the district and in nearby districts with similar characteristics and goals;
- recommendations or authorizations of state or provincial committees where they exist;
- independent information bases such as the National Diffusion Network; and,

- representatives of all publishers of appropriate material.

Instructional materials developed by not-for-profit organizations as well as commercial publishers should be considered. Whenever possible, committee representatives should examine materials displayed at a national or regional NCTM meeting. Interviews with members of the author team or a publisher's representative may also be helpful.

Once the materials for review have been collected, each should be evaluated for:

- content coverage and development of each major curricular strand;
- development of the full range of cognitive abilities, especially problem solving and analysis;
- correlation of the content with local, state or provincial, or national scope-and-sequence guidelines;
- correlation of the content and instructional approach with research on mathematics teaching and learning;
- correlation of the content and instructional approach with the standards of mathematics professional organizations;
- correlation between textual features and characteristics of special students;
- evidence of success based on publishers field validation studies, implementation in other districts, critical reviews in professional journals and reports, or local field testing where possible; and,
- representation of the diversity of national social, political, and economic life.

Since instructional materials comprise a small fraction of any school's budget, the cost of the materials should not be a factor. Potential financial savings do not justify the selection of less desirable materials. Where textbooks are accompanied by ancillary materials, however, it is important to assess the extent to which successful use of the text depends on such extra materials.

Standard 4:
Implementation of New Instructional Materials

The selection of new instructional materials must be only part of a careful curriculum development process that includes appropriate in-service preparation for teachers and supervisors and a formal evaluation of the new program's effectiveness.

In-service preparation should acquaint all teachers with the general program goals and the emphasis for each topic in every course. This emphasis should, where possible, be identified in terms of the appropriate number of instructional days to be devoted to each topic, and the topics should be correlated with specific portions of the new instructional materials. In-service should provide the additional mathematics background and preferred instructional approaches to the mathematics content.

Implementation should include periodic evaluation surveys of teachers' and students' reactions to the materials as well as an analysis of student achievement. Results of these evaluations should be used to evaluate the achievement of the program goals and plan for ongoing modification of the program. Implementation is typically more successful when such in-service support continues throughout the academic year.

Standard 5:
Professional Integrity

The instructional materials selection process must not allow influences that compromise the professional integrity of the process.

Any person connected with the process of selection and implementation of instructional materials who has a financial interest in any program being considered must make that interest known publicly at the outset. For the duration of the selection process those involved must not accept gifts of travel, entertainment or materials from a vendor.

The instructional materials recommended by the Selection Committee must be adopted unless irregularities in the selection process are documented.

(April 1984)

Provisions for Mathematically Talented and Gifted Students

All students deserve the opportunity to achieve their full potential; talented and gifted students in mathematics deserve no less. It is a fundamental responsibility of all school districts to identify mathematically talented and gifted students and to design and implement programs that meet their needs. Further, it is the responsibility of mathematics educators to provide appropriate instruction for such students.

The identification of mathematically talented and gifted students should be based on multiple assessment measures and should involve teachers, counselors, administrators, and other professional staff. In determining admission to talented and gifted programs, the evaluators must consider the student's total educational development as well as his or her mathematical ability, achievement, and aspirations. Eligible students and their parents should fully understand the nature and demands of the program before making a commitment to participate. Unqualified students should not be admitted for any reason.

The needs of mathematically talented and gifted students cannot be met by programs of study that only accelerate these students through the standard school curriculum, nor can they be met by programs that allow students to terminate their study of mathematics before their graduation from high school. The curriculum should provide for all mathematically talented and gifted students every year they are in school. These students need enriched and expanded curricula that emphasize higher-order thinking skills, nontraditional topics, and applications of skills and concepts in a variety of contexts.

Therefore, the National Council of Teachers of Mathematics recommends that all mathematically talented and gifted students should be enrolled in a program that provides a broad and enriched view of mathematics in a context of higher expectation. Acceleration within such a program is recommended only for those students whose interests, attitudes, and participation clearly reflect the ability to persevere and excel throughout the entire program.

(October 1986)

How Teachers can use the Arithmetic Teacher

By **Elaine M. Bologna,** *director of the Demonstration Mathematics Center for Grades 1–6, Summit School, Winston-Salem, North Carolina; member of the* Arithmetic Teacher *Editorial Panel*

Featuring an article on "how teachers can use the *Arithmetic Teacher*" might be construed as "bringing coals to Newcastle" because the *Arithmetic Teacher* is published *for* teachers. Nonetheless, we would like to offer our readers some suggestions to enhance an already valuable mathematics resource.

For instance, have you—

• made copies each month of the "From the File" items; glued them to 4-by-5 file cards; color coded, with pencils, the tab portion (red for "Computation," blue for "Fractions," and so on); laminated these cards; and then, placed them in a handy file box?

Even though some of the items may be on topics not included in the curriculum for your grade, don't discard them. Instead, think of ways to scale down (or extend) the idea to satisfy the developmental level of your students. Even very young children can develop intuitive ideas about percent and fractions.

• tried using an IDEAS activity in a different way? Try using one, with an overhead projector, as a whole-class activity. Many of the activities are excellent for spatial visualization (e.g., "IDEAS," April 1981 and November 1979). Following whole-class work, laminate the sheets for use by individual students. Children can write on these laminated sheets with wax crayon or overhead-transparency pens, which can be wiped or washed off. Thus, you can reuse these sheets with the same students or others.

• had students color the "IDEAS" posters, laminate them, and post them on the bulletin board (e.g., October 1980, color the pumpkins orange; and March 1980—color in a pattern)?

• tried making *two* copies of some "IDEAS" pages, laminating them, and then cutting them apart so that the figures can be used as a matching activity (e.g., April 1981 and September 1979)?

• taken an "IDEAS" sheet and made other examples of the same type of activity (e.g., November 1979—use large coloring-book pictures to make other activities or change the numerals or examples; February 1979—change numbers inside the shapes or change the secret number; November 1978—change the problems)?

• noted how often articles—"Let's Do It . . ." and "IDEAS"—have illustrations of various geometric shapes that make very accurate templates or tracing forms for yourself or children? Often they can be duplicated, pasted on cards, and used in card games (e.g., "IDEAS," April 1981 and October 1978; p. 18 and pp. 38–39, March 1981; pp. 19–20, April 1980; pp. 25–29, January 1979; p. 24, October 1978; p. 58, March 1978; IDEAS, December 1977).

• looked carefully at "Books and Materials" and "Reviewing and Viewing" for books that seem particularly appealing? Do you give the necessary ordering information to your librarian? Most school librarians are happy to have teachers recommend good books for the professional or school library.

• begun to make a file of good problems to use when teaching problem solving? Make copies, paste them on cards, laminate the cards, and then keep them in a file marked "Problem Solving" (e.g., "Doing Mathematics is Problem Solving," pp. 40–43, April 1981; "IDEAS," pp. 28–32, March 1981; "The Multiple Triangle: Experiences in Discovery," pp. 34–38, January 1979; "Problem Solving with Calculators," pp. 24–26, and "Let's Do It," pp. 6–11, April 1978; and "Let's Do It," pp. 6–10, November 1977.

• made a file of "quickie" activities that can be kept in a folder and used throughout the year for reinforcing skills (e.g., "Classroom Activities for Able Students: In Kindergarten, First, and Second Grades," p. 48, February 1981; "Back to Basics the Magical Way," pp. 22–26, September 1979; "Equation Hangman," p. 46, January 1979; "Teacher-made Aids for Teaching Place Value and Estimation," p. 50, January 1978)?

• noticed how many articles have excellent illustrations that can be used for teaching? Have you mounted these on cards, and then let a small group of students work through the pictures, arriving at a generalization of the concept involved? Or, have you given out some illustrations with pictured materials and allowed the students to construct and invent a game or use for material (e.g., "Let's Do It," p. 6–12, and "Painless Division with Doc Spitler's Magic Division Estimator," p. 26, March 1981; "Cross-Number Puzzles," p. 14, December 1980; "Using Geoboards in the Primary Grades," pp. 22–25, April 1980; "Fractions on Grid Paper," pp. 8–10, January 1979; "A Pocket Multiplier," p. 54, March 1978; "Star Patterns," pp. 12–14 and "Mr. Ten, You're Too Tall!" pp. 20–22, January 1978)?

• laminated any of the covers of the *Arithmetic Teacher,* and then pasted, on the reverse side, the suggestions for use in "By Way of Introduction" (i.e., January 1978)?

There is no doubt that the *Arithmetic Teacher* can be the classroom teacher's best source of ideas, not only to improve the curriculum but also to transform a routine classroom into an exciting learning laboratory. No matter how old the issue, it is never out of style.

You may think or know of other ways to make use of the *Arithmetic Teacher.* Why not pass your ideas on to other readers! "Readers' Dialogue" and "From the File" are both appropriate media for sharing good teaching ideas. ■

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March 15-17	NCTM Regional Conference in Chattanooga
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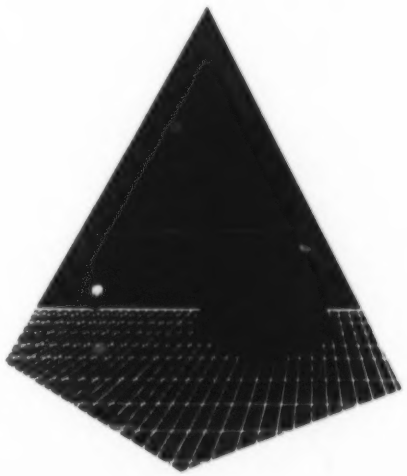

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